7.6 Bayes’ Theorem

Idea: IF we are given information about $P(E|F)$, what can we say about $P(F|E)$?

**Theorem 1: Bayes’ Theorem**

1. Let $S$ be a sample space partitioned into $n$ events, $A_1, A_2, ..., A_n$.

2. Let $E$ be an event in $S$

The probability of $A_i$, given $E$ has occurred

$$P(A_i|E) = \frac{P(A_i)P(E|A_i)}{P(E)}$$

$$= \frac{P(A_i)P(E|A_i)}{P(A_1)P(E|A_1) + P(A_2)P(E|A_2) + P(A_3)P(E|A_3) + \cdots + P(A_n)P(E|A_n)}$$

**MY WAY:**

$$P(A_i|E) = \frac{P(A_i \cap E)}{P(E)}$$
Example 1

Refer to the venn diagram. Use it to answer the following questions

1. \( P(D) = \frac{55}{100} = \frac{11}{20} \)

2. \( P(A|D) = \frac{20}{55} = \frac{4}{11} \)

3. \( P(D^c) = \frac{45}{100} = \frac{9}{20} \)

4. \( P(B|D^c) = \frac{10}{45} = \frac{2}{9} \)

Draw a tree diagram

\[
P(D) = P(A \cap D) + P(B \cap D) + P(C \cap D)
\]

\[
P(D) = \frac{P(A \cap D)}{P(D|B) \cdot P(B)}
\]
Example 3: Use class survey from 7.5

In a survey of 109 students, it was found that 66 were female. Additionally, it was found that 77% of those females play games, whereas 83% of the males play games. Find the probability a randomly selected student:

1. is female given they play games?

\[
P(F \mid G) = 0.77
\]

\[
P(G \mid M) = 0.83
\]
Example 2

210 M&Ms are in a bag (140 green, 70 blue). I currently have in my lunchbox 12 green and 4 blue. A M&M is drawn at random from the bag and placed in my lunch box. During lunch I randomly select a blue M&M. What is the probability the transferred M&M from the bag was green?

\[ P(\text{green bag} \mid \text{blue lunch}) = \frac{P(\text{green bag} \cap \text{blue lunch})}{P(\text{blue lunch})} \]

\[ = \frac{\frac{140}{210} \times \frac{4}{17}}{\frac{140}{210} \times \frac{4}{17} + \frac{20}{210} \times \frac{5}{17}} \]

\[ = \frac{8151}{8151} \]

\[ = \frac{8}{13} \text{ or } 0.61 \]