6.2 Number of Elements

- Determined by counting the all the elements in the set
- Denoted by \( n(A) \)

**Example 1**

Let \( A = \{0, -1, 5, 7\} \), \( B = \{0, 1, 2, 3, ..., 15\} \), \( C = \emptyset \), and \( D = \{x \mid x \text{ is a letter of the alphabet}\} \). Find

1. \( n(A) = 4 \)
2. \( n(B) = 16 \)
3. \( n(C) = 0 \)
4. \( n(D) = 26 \)
5. \( n(B \cup D) = 16 + 26 = 42 \)

6. \( n(A \cap B) \)

\[ A \cap B = \{0, 5, 7\} \]

7. \( n(A \cup B) \)

\[ A \cup B = \{0, -1, 5, 7, 1, 2, 3, ..., 15\} \]

**Formula 1**

For any two sets \( A \) and \( B \)

\[
\quad n(A \cup B) = n(A) + n(B) - n(A \cap B)
\]

\#7 Again

\[ = 4 + 16 - 3 = 17 \]
Example 2

In a survey of 100 coffee drinkers, it was found that 70 take sugar, 60 take cream, and 50 take both.

1. How many coffee drinkers take sugar or cream? \[ 80 = 20 + 50 + 10 \]

\[
\begin{align*}
\text{n}(S \cup C) &= \text{n}(S) + \text{n}(C) - \text{n}(S \cap C) \\
&= 70 + 60 - 50 = 80
\end{align*}
\]

2. How many take neither sugar nor cream?

VENN DIAGRAM IS BEST

\[ S^c \cap C^c \]

20 took neither
Example 3

In a recent survey of 200 people, 100 of them indicated they subscribe to Hulu, 60 indicated they subscribe to Netflix, and 40 indicated they subscribe to both.

\[ n(H \cup N) = n(H) + n(N) - n(H \cap N) \]
\[ = 100 + 60 - 40 \]
\[ = 120 \]

1. How many people subscribe to at least one service?

2. How many subscribe to ONLY Hulu?

\[ n(H \cap N^c) = 60 \]

3. How many subscribe to exactly one of the services?

**ONLY HULU OR ONLY NETFLIX**

\[ 60 + 20 = 80 \]

4. How many subscribe to neither service?

\[ 80 \]

\[ \text{DON'T SUBSCRIBE TO HULU?} \quad n(H^c) = 100 \]

\[ \text{OR} \quad 200 - n(H) = 200 - 100 \]
\[ = 100 \]
**Formula 2**

For any three sets $A$, $B$, and $C$

\[
\begin{align*}
\quad n(A \cup B \cup C) &= n(A) + n(B) + n(C) - n(A \cap B) \\
&\quad - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)
\end{align*}
\]

**Example 4**

If $n(A) = 12$, $n(B) = 12$, $n(A \cap B) = 5$, $n(B \cap C) = 4$, $n(A \cap C) = 5$, $n(A \cap B \cap C) = 2$, and $n(A \cup B \cup C) = 25$. Find $n(C)$

\[\begin{align*}
\quad 4 + 3 + 5 + 3 + 2 + 2 + x &\quad = 25 \\
\quad 16 + x &\quad = 25 \\
\quad x &\quad = 9
\end{align*}\]

\[\begin{align*}
\quad 25 &= 12 + 12 + n(C) - 5 - 4 - 5 \\
\quad 25 &= 12 + n(C) + 2 \\
\quad 25 &= 12 + n(C) \\
\quad n(C) &= 13
\end{align*}\]

**Example 5**

Let $A$ and $B$ be subsets of $U$ and suppose $n(U) = 200$, $n(A) = 100$, $n(B) = 80$, and $n(A \cap B) = 40$. Compute

1. $n(A^c)$
2. $n(a \cap B^c)$
Example 6

A survey of 50 students was conducted and the following results were obtained:

- 20 like math
- 35 like science
- 16 like English
- 13 like math and science
- 5 like math and English
- 7 like science and English
- 3 like all three

1. How many students like at least one of the subjects?

\[ 45 + 14 + 35 + 8 + 33 + 10 + 17 = 162 \]

2. How many students liked none of the subjects?

3

3. How many liked math and science but not English?

14

4. How many liked math or English?

\[ 45 + 14 + 35 + 8 + 33 + 10 = 145 \]