5.3 Amortization and Sinking Funds

**Definition 1: Amortization Formula**

The periodic payment \( R \) on a loan of \( P \) dollars to be amortized over \( n \) periods with interest charged at the rate of \( i \) per period is

\[
R = \frac{P \cdot i}{1 - (1+i)^{-n}}
\]

**Example 1**

A car dealership offers 2.5% financing over a 5 year loan period for a car priced at $10,000. The other option is a $700 rebate but you must finance at 4% over 5 years. Which option is better?

\[
P = 10,000, \ r = .025, \ + = 5, \ m = 12
\]

2.5%:

\[
R = \frac{10,000 \cdot \left( .025 \right)}{1 - (1 + .025/12)^{-60}} = \frac{20,833.333}{.17388341} = \$171.47
\]

700 REBATE:

\[
R = \frac{9300 \cdot \left( .04 \right)}{1 - (1 + .04/12)^{-60}} = \frac{31}{.180996896} = \171.270
\]
Example 2

Mike purchased a house for $200,000, put down 10%, and financed the remaining balance with a 5% interest over 30 years. What should the monthly mortgage payment be? How much did Mike pay in interest?

10% of 200,000 is $20,000. Loan amount is $180,000

Know: $t = 30, r = 0.05, m = 12, P = 180,000$

\[ R = \frac{180,000 \left( 0.05 \div 12 \right)}{1 - \left( 1 + 0.05 \div 12 \right)^{-360}} = \frac{750}{0.776173404} = \$966.28 \]

Overall Mike pays $966.28 \times 360 = $347,860.80 for a 180,000 loan

Interest = $347,860.80 - 180,000

= $167,860.80
Example 3

Brian needs to save $150,000 to pay off a debt in 10 years. He will pay monthly payments into an account earning 4% compounded quarterly. What size payment should he make?

\[ R = \frac{S \cdot \frac{i}{1+i}}{(1+i)^n-1} \]

\[ R = \frac{150000 \cdot \frac{.04}{4}}{(1+.04/4)^{40} - 1} = \frac{1500}{.0048856323} = 3068.34 \]