5.2 Annuities

**Definition 1: Annuity**

A **SEQUENCE OF PAYMENTS**

**Definition 2: Future Value of an Annuity**

The future value $S$ of an annuity of $n$ payments of $R$ dollars earning interest rate of $i$ per period is

$$S = R \left[ \frac{(1+i)^n - 1}{i} \right]$$

$i = \frac{r}{m}$

$n = m \cdot t$

**Example 1**

John will need $12,000 for a down payment in three years. He deposits $200 per month earning 5% per year compounded monthly. Will he have enough?

$$R = 200, \quad r = .05, \quad m = 12, \quad i = \frac{.05}{12}$$

$$S = 200 \left[ \frac{(1+\frac{.05}{12})^{2.3} - 1}{\frac{.05}{12}} \right] = \frac{1614722313}{.00416667}$$

$$= 200 \left( \frac{38.753305}{.05} \right) = \frac{7750.66}{200 \times 36 = \$7200}$$

**NOT ENOUGH**

**INTEREST EARNED**

$7750.66 - 7200 = \$550.66$
Example 2

Suppose John consulted you on what he would have to save per month so he has $12,000 in three years. At an interest rate of 5% per year compounded monthly, what would he save to deposit per month?

\[ S = R \left[ \frac{(1+i)^n-1}{i} \right] \]

\[ 12000 = R \left[ \frac{(1+.05/12)^{2.3} - 1}{.05/12} \right] \]

\[ 12000 = R \left( 38.753305 \right) \]

\[ \frac{12000}{38.753305} = R \]

\[ \$ 309.65 = R \]

Definition 3: Present Value of an Annuity

The present value of \( P \) of an annuity consisting of \( n \) payments of \( R \) dollars each, earning interest at \( i \) per period is:

\[ P = R \left[ \frac{1 - (1+i)^n}{i} \right] \]
Example 3

Find the present value of an annuity consisting of 24 quarterly payments of $250 each and earning 3% per year compounded quarterly.

\[
P = 250 \left( \frac{1 - (1 + 0.03/4)^{4 \times 24}}{0.03/4} \right)
\]

\[
= 250 \left( \frac{0.164166}{0.0075} \right)
\]

\[
= 5472.29
\]

Example 4

Suppose you’re 22 years old, just graduated from college, and begin thinking of retirement. There are many options out there. Which option is better?

1. You get a job and deposit $150 per month into an account earning 5% per year compounded monthly for 7 years. You leave this money alone until the age of 65.

2. It’s too hard to save when you’re 22. You wait until you’re 45 years old and then deposit $150 per month at 5% per year compounded monthly. How much will you have when you’re 65 years old?

1) After 7 years: \( S = 150 \left( \frac{(1 + 0.05/12)^{12 \times 7} - 1}{0.05/12} \right) = $5,049.30 \)

Let it sit for 36 years → use \( P(1+r)^n \)

\[
150 \times 12 \times 7 = $12,600
\]
\[
(2) \quad S = 150 \left[ \frac{(1 + 0.05/12)^{12 \cdot 20} - 1}{0.05/12} \right] = \$61,655.05
\]

Spent $150 \times 12 \times 20 = \$36000$
Example 5

Brian payed a down payment of $12,000 towards a new car. He secured a loan for 60 months at an interest rate of 1.99% per year compounded monthly. His monthly payments are $232 per month. How much was the car worth?

\[
\text{CAR PRICE} = \text{PRESENT VALUE OF \ LOAN} + \text{DOWN PAYMENT} \downarrow \uparrow$12,000
\]

\[n = 60 \quad \gamma = 0.0199 \quad m = 12 \quad R = \$232\]

\[
P = R \left( \frac{1 - (1 + \frac{i}{m})^{-n}}{\frac{i}{m}} \right) \quad i = \frac{\gamma}{m}
\]

\[
= 232 \left[ \frac{1 - (1 + \frac{0.0199}{12})^{-125}}{(0.0199/12)} \right]
\]

\[
= 232 \left[ \frac{0.09463544}{0.00165833} \right]
\]

\[
= 232 \left[ 57.0667117 \right]
\]

\[
\text{LOAN AMOUNT} = 13,239.48 \quad \text{CAR PRICE} = 13,239.48 + 12,000 = \$25,239.48
\]