5.1 Compound Interest

**Definition 1: Principle**

INITIAL AMOUNT

**Definition 2: Simple Interest**

YEARLY CHARGE, ONLY ON ORIGINAL PRINCIPLE

**Definition 3: Simple Interest Formulas**

\[
I = \text{INTEREST}, \quad P = \text{PRINCIPLE}, \quad t = \text{TIME}, \quad \gamma = \text{ANNUAL INTEREST RATE}
\]

\[
I = Prt
\]

\[
A = P(1+\gamma t) \quad \text{A FUTURE BALANCE}
\]

**Example 1**

A bank pays simple interest at the rate of 8% per year. If a customer deposits $1000 and makes no withdrawals for 3 years, what is the total amount at the end of 3 years?

What is the interest earned?

\[
I = Prt = 1000(0.08)(3) = 240
\]

TOTAL AMOUNT AFTER 3 YEARS IS $1240
Example 2

Suppose you buy a TV for $1500 that is advertised at $30 a month for 5 years. How much did you pay in interest?

You paid $30 \times 60 = $1800

But TV costs $1500, so interest earned is $300

Example 3: Compound Interest

You put $5000 in an account that pays 4% compound interest per year.

1. At the end of the 1st year:

\[ A = 5000(1 + 0.04) = 5000(1.04) = 5200 \]

Earned $200

2. At the end of the 2nd year:

\[ A = 5200(1 + 0.04) = 5200(1.04) = 5408 \]

Earned $208

3. At the end of the 3rd year:

\[ A = 5408(1 + 0.04) = 5408(1.04) = 5624.32 \]

Earned $216.32
**Definition 4: Compound Interest Formula**

\[ A = P(1+i)^n, \quad i = \frac{r}{m}, \quad n = mt \]

- **A** = Accumulated (Future) Amount
- **P** = Principal
- **r** = Nominal Interest Rate (Given)
- **m** = # of Compounding Periods Per Year
- **t** = # of Years

**Example 4**

If $3000 is saved with an interest rate of 2% per year. Find the accumulated amount after 3 years if compounded

1. annually \( m = 1 \) \[ P = 3000 \]
2. semiannually \( m = 2 \) \[ r = 0.2 \]
3. quarterly \( m = 4 \) \[ t = 3 \text{ years} \]
4. monthly \( m = 12 \)
5. weekly \( m = 52 \)

(1) **Annually:** \[ A = 3000 \left(1 + \frac{0.02}{1}\right)^{1 \cdot 3} = 3183.62 \]

(2) **Semi-Annually:** \[ A = 3000 \left(1 + \frac{0.02}{2}\right)^{2 \cdot 3} = 3184.56 \]

(3) **Quarterly:** \[ A = 3000 \left(1 + \frac{0.02}{4}\right)^{4 \cdot 3} = 3185.03 \]
(4) **MONTHLY**  \[ A = 3000 \left(1 + \frac{0.2}{12}\right)^{12 \cdot 3} = 3185.35 \]

(5) **WEEKLY:**  \[ A = 3000 \left(1 + \frac{0.2}{52}\right)^{52 \cdot 3} = 3185.47 \]
Example 5

What is the balance on a principle amount of $3500, saved at 3% compounded monthly after (a) 1 month, (b) 1 year, (c) 10 years, (d) 40 years

(a) 1 MONTH $A = 3500 \left( 1 + \frac{0.03}{12} \right)^{12 \cdot \frac{1}{12}} = \$3508.75$

(b) 1 YEAR: $A = 3500 \left( 1 + \frac{0.03}{12} \right)^{12 \cdot 1} = \$3606.46$

(c) 10 YEARS $A = 3500 \left( 1 + \frac{0.03}{12} \right)^{12 \cdot 10} = \$4722.74$

(d) 40 YEARS $A = 3500 \left( 1 + \frac{0.03}{12} \right)^{12 \cdot 40} = \$11,603.02$
Example 6

You deposit $2000. Suppose the stated interest rate (nominal) is 4% compounded monthly. If you were to receive interest ONCE, what would the interest rate have to be so that you earn the same amount?

MONTHLY: \[ A = 2000 \left( 1 + \frac{.04}{12} \right)^{12 \cdot 1} = 2081.48 \]

ONCE: \[ \frac{2081.48}{2000} = 1 + r \]
\[ 1.0407415 = 1 + r \]
\[ .0407415 = r \]
\[ r = 4.074\% \]

or \( r = \frac{4.074\%}{APY} \)

Definition 5

\[ \text{APY} - \text{EFFECTIVE INTEREST RATE} \]

\[ r_{eff} = \left( 1 + \frac{r}{m} \right)^m - 1 \]

\[ \text{APY} \]
Example 7

Find the effective rate of interest corresponding to a nominal rate of 4% per year compounded (a) annually, (b) semiannually, (c) quarterly, (d) daily

1. Annually

\[(1 + 0.04/1)^1 - 1 = 0.04 \text{ or } 4\%
\]

2. Semiannually

\[(1 + 0.04/2)^2 - 1 = 0.0404 \text{ or } 4.04\%
\]

3. Quarterly

\[(1 + 0.04/4)^4 - 1 = 0.0406 \text{ or } 4.06\%
\]

4. Daily

\[(1 + 0.04/365)^{365} - 1 = 0.0408 \text{ or } 4.08\%
\]

Definition 6: Present Value

\[P = A \left( 1 + i \right)^{-n}
\]

\[\bar{t} = \frac{T}{n}
\]

\[n = m +
\]
Example 8

Suppose that you want to take a trip to Europe in 6 years and figure you need $7500. To have that much set aside in 6 years, how much do you have to deposit today into a bank earning 5% compounded quarterly?

\[ P = A (1 + i)^{-n} \]

\[ P = 7500 \left(1 + \frac{.05}{4}\right)^{-4 \cdot 6} \]

\[ = \$5,566.48 \]