1.5 Composition of Functions

**Definition 1.3** (Composition of Functions). For functions $f(x)$ and $g(x)$, the composite function $f \circ g$ is

$$f \circ g = f(g(x))$$

People struggle with composition for many reasons. Let’s go through a series of examples that will build up to the definition we just stated for the composition of functions.

<table>
<thead>
<tr>
<th>Function</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>$2x^2 - \cos(x) + 4x - 3$</td>
</tr>
<tr>
<td>$f(2)$</td>
<td>$2(2)^2 - \cos(2) + 4(2) - 3$</td>
</tr>
<tr>
<td>$f(-6)$</td>
<td>$2(-6)^2 - \cos(-6) + 4(-6) - 3$</td>
</tr>
<tr>
<td>$f(a)$</td>
<td>$2(a)^2 - \cos(a) + 4(a) - 3$</td>
</tr>
<tr>
<td>$f(2x + 1)$</td>
<td>$2(2x + 1)^2 - \cos(2x + 1) + 4(2x + 1) - 3$</td>
</tr>
<tr>
<td>$f(g(x))$</td>
<td>$2(g(x))^2 - \cos(g(x)) + 4(g(x)) - 3$</td>
</tr>
</tbody>
</table>

All of these are composition of functions. The last two are more obvious. A composition of functions is just evaluating a function, like $f(2)$. The difference is now we evaluate a function with another function. So $f(2x + 1)$ is a composition of functions. Remember, all you’re doing is plugging one function into another, just like how you would evaluate any function.

**Example 1.9.**

1. Let $f(x) = 3x^2 - 4x + 1$ and $g(x) = 3x - 5$. Find

   (a) $f \circ g$:

   First, always rewrite $f \circ g$ as $f(g(x))$. 

21
So, \( f \circ g = f(g(x)) = f(3x-5) = 3(3x-5)^2 - 4(3x-5) + 1 \).

(b) \((f \circ g)(2)\). This notation is really just asking for \( f(g(2)) \).

\[ f(g(2)) = f(1) = 3(1)^2 - 4(1) + 1 = 0 \]

2. Let \( f(x) = \frac{1}{x} \) and \( g(x) = x + 1 \). Find the domain of

(a) \( f \circ g \):

Let’s just take a look at what \( f(g(x)) \) is.

\[ f(g(x)) = f(x+1) = \frac{1}{x+1} \]

There are two ways of doing this. One way is just to find \( f \circ g \) (do NOT simplify it) and simply find it’s domain. From above you can see \( x \neq -1 \). Another way is to do it in steps. First, we look at the domain of \( g(x) \). Since \( g(x) = x + 1 \), we have no domain issues. Ok, so we can plug anything into \( g(x) \), but what about \( f(x) \). Notice that we can’t plug 0 into \( f(x) \). And what do I plug into \( g(x) \) that will give me \( g(x) = 0 \)?

(b) \( g \circ f \):

\[ g(f(x)) = g\left(\frac{1}{x}\right) = \frac{1}{x} + 1 \]

We can never have a denominator equal 0, so \( x \neq 0 \). So the domain is all reals except \( x = 0 \).