Math 210 Sample Final Exam

1. You wish to purchase a house. You have a down-payment of \$12,000 and can afford a mortgage of \$900 per month. You find a 30 year loan that charges 7.2% compounded monthly. What price can you afford for your house?

(a) \$228,431
(b) \$359,328
(c) \$126,307
(d) \$144,589
(e) \$324,000

USE $P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$ → LOAN AMOUNT

$= 900\left[\frac{1-(1+.072/12)^{-360}}{(.072/12)}\right]$

$= 132{,}589$

PRICE = DOWN PAYMENT + LOAN

$= 12000 + 132{,}589$

$= \$144{,}589$

2. Mom and Dad and their 4 children line up for a picture. How many different pictures can be taken if Mom and Dad do not have the four children between them?

(a) 964
(b) 480
(c) 240
(d) 672
(e) 124

1) ANSWER = TOTAL POSSIBLE ARRANGEMENTS − ARRANGEMENTS WHERE MOM AND DAD HAVE ALL KIDS BETWEEN THEM

2) $2 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 1 = 48$ POSSIBILITIES WHERE CHILDREN ARE BETWEEN PARENTS

3) $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$ TOTAL POSSIBILITIES

3. Maximize $P = 2x + 10y + 6z$ subject to the constraints:

$$\begin{array}{rcl} x + 2y & \leq & 10 \\ y + 3z & \leq & 24 \\ x, y, z & \geq & 0 \end{array}$$

$x + 2y + s_1 = 10$
$y + 3z + s_2 = 24$
$-2x - 10y - 6z + P = 0$

The optimal solution is:

(a) $x = 44,\ y = 0,\ z = 0.$
(b) $x = 0,\ y = 5,\ z = 19/3.$
(c) $x = 19/3,\ y = 0,\ z = 5.$
(d) $x = 5,\ y = 19/3,\ z = 0.$
(e) $x = 0,\ y = 19/3,\ z = 5.$

(1)

x	y	z	s_1	s_2	P	C
1	2	0	1	0	0	10
0	1	3	0	1	0	24
-2	-10	-6	0	0	1	0

(2)

x	y	z	s_1	s_2	P	C
1/2	1	0	1/2	0	0	5
0	1	3	0	1	0	24
-2	-10	-6	0	0	1	0

(3) $R_2 - R_1$
$\frac{1}{3}R_2 + 10R_1$

x	y	z	s_1	s_2	P	C
1/2	1	0	1/2	0	0	5
-1/2	0	3	-1/2	1	0	19
3	0	-6	5	0	1	50

#3 CONT.

$\frac{1}{3}R_2 \rightarrow$

x	y	z	S_1	S_2	p	c
1/2	1	0	1/2	0	0	5
-1/6	0	(1)	1/6	1/3	0	19/3
3	0	-6	5	0	1	50

$R_3 + 6R_2 \rightarrow$

x	y	z	S_1	S_2	p	c
1/2	1	0	1/2	0	0	5
-1/6	0	1	-1/6	1/3	0	19/3
2	0	0	4	2	1	88

4. Which entry is a possible pivot in the following simplex tableau?

$$\left[\begin{array}{cccccc|c} 15 & 1 & -3 & 1 & 0 & 0 & 6 \\ -2 & -3 & 4 & 0 & 1 & 0 & -50 \\ \hline -2 & 3 & 4 & 0 & 0 & 1 & 0 \end{array}\right]$$

(a) 15.
(b) -2.
(c) 1.
(d) -3.
(e) None of the above.

POSSIBLE PIVOTS ARE 15 OR 1

5. In solving a system of linear equations using Gauss-Jordan elimination, the following augmented matrix is obtained:

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 6 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow \begin{array}{l} x = 6 + 3z \\ y = 7 - 2z \\ z = \text{ANYTHING} \end{array}$$

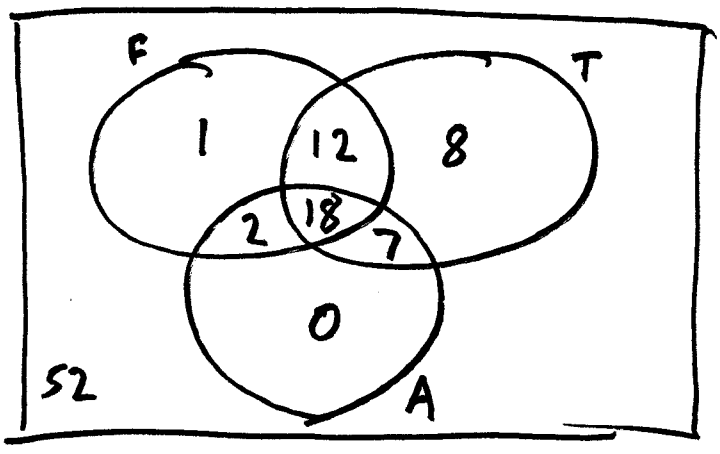
Which is true?

(a) There are no solutions for this problem.
(b) There are infinitely many solutions, one of which is $x = 9,\ y = 5, z = 1$.
(c) There is exactly one solution and $x = 6,\ y = 7$, and $z = 0$.
(d) There are infinitely many solutions, one of which is $x = 0,\ y = 11,\ z = 2$.
(e) There are exactly two solutions for this problem.

NOT (d) BECAUSE THIS SOLUTION DOESN'T FIT

6. In a shipment of 100 cars from Japan:
33 have a four-cylinder engine, F
45 have tilt steering wheels, T
27 have air conditioning, A
30 have both four-cylinder engines and tilt steering wheels,
25 have tilt steering wheels and air conditioning,
18 have four-cylinder engines, tilt steering wheels and air conditioning,
20 have air conditioning and a four-cylinder engine. How many cars have air conditioning and neither tilt steering wheels nor four-cylinder engines?

(a) 4
(b) 0
(c) 3
(d) 5
(e) None of the above.



7. Jack and Sally want to buy a house that costs $210,000. They put $20,000 down and get a 20 year loan at 7.8%. What will their monthly mortgage payment be?

(a) $3748

(b) $1349

(c) $1730

(d) $1566

(e) $792

LOAN AMOUNT = 210,000 − 20,000 = 190,000

USE: $R = \frac{Pi}{1-(1+i)^{-n}}$ $P = 190{,}000$, $m = 12$, $n = 240$, $r = 0.078$, $t = 20$

$$R = \frac{190000(.078/12)}{1-(1+.078/12)^{-240}} = \$1566$$

8. Solve the system :

$$\begin{aligned} x - y + 5z &= 13 \\ y - 2z &= -7 \\ y + 8z &= 33 \end{aligned}$$

CLASSIC WAY TO SOLVE IS TO ROW REDUCE $\left[\begin{array}{ccc|c} 1 & -1 & 5 & 13 \\ 0 & 1 & -2 & -7 \\ 0 & 1 & 8 & 33 \end{array}\right]$

TO GET $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 4 \end{array}\right]$

$x = -6, y = 1, z = 4$

(a) In the solution: $x = -6$.

(b) In the solution: $x = 1$.

(c) There are infinitely many solutions.

(d) No solution; the equations are inconsistent.

(e) In the solution: $x = -3$.

9. 1000 tickets are sold earning $5100. Adult tickets cost $6 and childrens cost $4. How many adult tickets were sold?

(a) 550

(b) 450

(c) 620

(d) 470

(e) 320

SET UP SYSTEM OF EQUATIONS

	x	y	
TICKETS SOLD	1	1	1000
MONEY MADE	6	4	5100

$\begin{cases} x + y = 1000 \\ 6x + 4y = 5100 \end{cases}$ SOLVE THIS

$x = 550$

$y = 450$

x = ADULT TICKETS

y = CHILDREN TICKETS

10. A box contains 12 eggs of which 4 are rotten. If six are selected at random, what is the probability that at least one is rotten?

(a) 32/33

(b) 1/3.

(c) 14/33.

(d) 1/33.

(e) None of the above.

4 ROTTEN, 8 GOOD

ANSWER: $1 - P(\text{NONE ARE ROTTEN})$

$$1 - \frac{C(8,6)C(4,0)}{C(12,6)}$$

$$= 1 - \frac{28 \cdot 1}{924} = \frac{32}{33}$$

11. Minimize $C = 5x + 7y - z$ subject to the constraints:

$$\begin{aligned} 2x + y + z &\leq 12 \\ x, y, z &\geq 0 \end{aligned}$$

TRICK: CORNERS ARE (6,0,0) → 35, (0,12,0) → 84, (0,0,12) → -12. CHECK THESE WITH $C = 5x + 7y - z$

The optimal solution is:

(a) $x = 7,\ y = 8,\ z = 0.$

(b) $x = 0,\ y = 0,\ z = 24.$

(c) $x = 5,\ y = 7,\ z = 0.$

(d) $x = 0,\ y = 0,\ z = 12.$

(e) None of the above.

12. The line $3x + 4y = 24$ has x-intercept:

(a) 3

(b) 4

(c) 8

(d) 24

(e) None of the above.

↑ MEANS $y = 0$

$3x + 4(0) = 24$

$3x = 24$

$x = 8$

13. Amongst families with four children; are the events "At least one boy" and "At least one boy and at least one girl" independent?

(a) No

(b) Yes

(c) Neither of the above is true

(d) All of the above are true

(e) None of the above.

THIS COULD BE DONE A FEW WAYS. HERES WHAT I DID

1) LET A = AT LEAST ONE BOY

LET B = AT LEAST ONE BOY AND ONE GIRL

2) IF ITS INDEPENDENT THEN $P(A|B) = P(A)$

3) $P(A) = 15/16$ BUT $P(A|B) = 1$ ← IF B IS TRUE THEN A IS ALWAYS TRUE

14. If I put \$5000 into an account earning 6% interest compounded monthly, how much will I have after 7 years?

(a) \$7602

(b) \$7100

(c) \$3288

(d) \$8493

(e) \$6431

USE $A = P(1+i)^{\wedge}$ $r = .06$, $m = 12$, $t = 7$

$A = 5000\left(1 + \frac{.06}{12}\right)^{12 \cdot 7}$

15. Fred puts \$75 each month into an account earning 6% interest. How much will he have after 15 years?

(a) \$21,811

(b) \$25,650

(c) \$13,500

(d) \$17,962

(e) \$33,130

USE $S = R\left[\frac{(1+i)^{\wedge}-1}{i}\right]$ $R = 75$, $r = .06$, $m = 12$, $t = 15$

$S = 75\left[\frac{(1+.06/12)^{180}-1}{(.06/12)}\right] = \$21,811$

16. If Pr(E)=1/12, Pr(E ∪ F)=1/6, and E and F are mutually exclusive events then Pr(F)=

(a) 1/11.

(b) 1/72.

(c) 1/24.

(d) 1/6.

(e) None of the above.

(1) IF E AND F ARE MUTUALLY EXCLUSIVE THEN $P(E \wedge F) = 0$

(2) USE $P(E \vee F) = P(E) + P(F) - P(E \wedge F)$

$1/6 = 1/12 + P(F) - 0 \rightarrow P(F) = 1/12$

17. Find the inverse of

$$A = \begin{pmatrix} 1 & 0 & 5 \\ 0 & 5 & 6 \\ 0 & 5 & 7 \end{pmatrix}$$

EITHER USE CALCULATOR DIRECTLY BY TYPING IN A^{-1} OR SOLVE $[A|I]$ BY ROW REDUCING

The entry in the third row and third column is:

(a) 0.

(b) 1/7.

(c) -1.

(d) 1

(e) None of the above.

$$A^{-1} = \begin{pmatrix} 1 & 5 & -5 \\ 0 & 7/5 & -6/5 \\ 0 & -1 & 1 \end{pmatrix}$$

ANSWER

18. What is the effective interest rate for an account making 7.5% compounded weekly? (i.e. 52 times a year)

(a) 7.78%

(b) 3.9%

(c) 7.63%

(d) 43.0%

(e) 8.23%

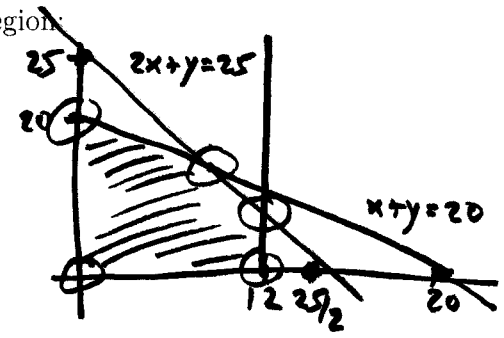
USE $r_{eff} = \left(1 + \frac{r}{m}\right)^{m} - 1$ $m = 52$, $r = .075$

$r_{eff} = \left(1 + \frac{.075}{52}\right)^{52} - 1 = .0778$

OR 7.78%

19. Which of the following is *not* a corner point of the region:

$$\begin{cases} 2x+y & \leq & 25 \\ x+y & \leq & 20 \\ x & \leq & 12 \\ x,y & \geq & 0 \end{cases}$$



(a) (5,15)
(b) (12,1)
(c) (12,0)
(d) (0,25)
(e) (0,0)

CORNERS ARE (0,20) (12,1) (0,0) (12,0) (5,15)

20. A company produces two models of fax machines: a Value and a Deluxe. Each Value model costs \$200 to make while each Deluxe costs \$300 to make.The profits are \$25 for each Value model and \$40 for each Deluxe. The total number of fax machines demanded per month does not exceed 2500 and the company has a budget of \$600,000 for manufacturing costs. How many units of each model should the company produce in order to maximize its profits? Suppose that x Value models and y Deluxe models are produced per month.

The maximum profit is :

(a) \$75,000
(b) \$77,500
(c) \$17,000
(d) \$80,000
(e) None of the above.

	x	y	
COST	200	300	600,000 → $200x+300y \leq 600000$
CAN'T EXCEED	1	1	2500 → $x+y \leq 2500$

MAXIMIZE $P = 25x+40y$

GO TO NEXT PAGE

21. In how many ways can the positions of President, Vice-President and Secretary of a club be filled if they are to be chosen from the 15 members and no member can hold more than one position?

(a) 42.
(b) 3.
(c) 2730.
(d) 15.
(e) 3375.

3 POSITIONS, 15 MEMBERS, ORDER MATTERS

OF WAYS IS $P(15,3) = 2,730$

#20 CONT. SOLVE MAXIMIZE $P = 25x + 40y$

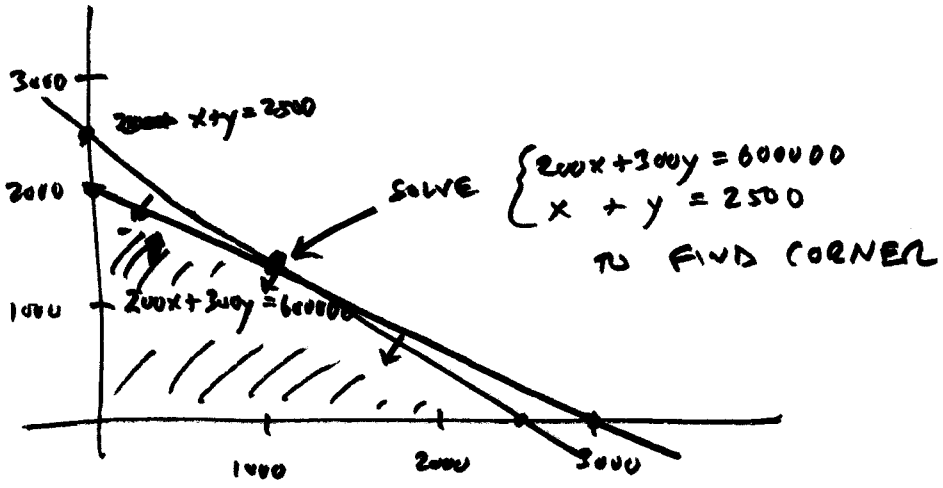
SUBJECT TO $200x + 300y \leq 600000$

$x + y \leq 2500$

$x, y \geq 0$

(1) CAN USE SIMPLE METHOD

(2) IE SOLVE BY CORNER METHOD SINCE IT ONLY HAS TWO VARIABLES



CORNERS ARE

(0,0) → $0 PROFIT

(0, 2000) → $80,000 PROFIT

(1500, 1000) → $77,500 PROFIT

(2500, 0) → $62,500 PROFIT

22. If Pr(E)=1/12, Pr(E ∪ F)=1/6, and E and F are independent events then Pr(F)=

(a) 1/11

(b) 1/6.

(c) 1/72.

(d) 1/24.

(e) None of the above.

(1) IF E AND F ARE INDEPENDENT THEN $P(E \cap F) = P(E) \cdot P(F)$

(2) FORMULA: $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$P(E \cup F) = P(E) + P(F) - P(E) \cdot P(F)$

$1/6 = 1/12 + P(F) - \frac{1}{12} P(F)$

$1/12 = \frac{11}{12} P(F) \rightarrow P(F) = 1/11$

23. Solve the system:

$$\begin{array}{rcrcrcl} x & + & y & - & 2z & = & 3 \\ 2x & - & 3y & + & 3z & = & 2 \\ 5x & - & 5y & + & 4z & = & 6 \end{array}$$

USE CALCULATOR TO ROW REDUCE

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & 3 \\ 2 & -3 & 3 & 2 \\ 5 & -5 & 4 & 6 \end{array}\right]$$

(a) In the solution $x = 1$.

(b) The system is inconsistent.

(c) In the solution $x = 2$.

(d) In the solution $x = 3$.

(e) In the solution $x = 4$.

LAST ROW [0 0 0 | 1] MEANS THERE IS NO SOLUTION "INCONSISTENT"

TO GET $\left[\begin{array}{ccc|c} 1 & 0 & -.6 & 0 \\ 0 & 1 & -1.4 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

24. Which of the following is true if $A = \{1, 3, 5, 8, 9, 10\}$ and $B = \{2, 4, 6, 8, 10\}$?

(a) A and B are disjoint. FALSE. THEY SHARE 10

(b) $A \cap B = \{1, 2, 3, 4, 5, 6, 8, 9, 10\}$ FALSE $A \cap B = \{8, 10\}$

(c) $A^c \cap B = \{1, 3, 5, 9\}$ FALSE

(d) $A \cup B = \{8, 10\}$ FALSE

(e) $8 \in A \cap B^c$ TRUE SINCE $A \cap B^c = \{1, 3, 5, 9\}$

25. Urn I contains three balls, one red and two blue. Urn II contains five balls, two red and three green. An urn is chosen at random and a ball is selected from it.

What is the probability that the ball is red?

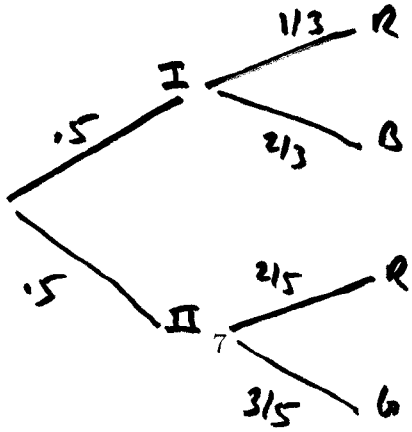
(a) 1/2.

(b) 11/15.

(c) 11/30.

(d) 1/3.

(e) 2/5.



$P(R) = P(I \cap R) + P(II \cap R)$

$= (.5)(1/3) + (.5)(2/5)$

$= 11/30$