MATH 210 FINITE MATHEMATICS

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7.6 Bayes' Theorem

Idea: IF we are given information about $P(E|F)$, what can we say about $P(F|E)$?

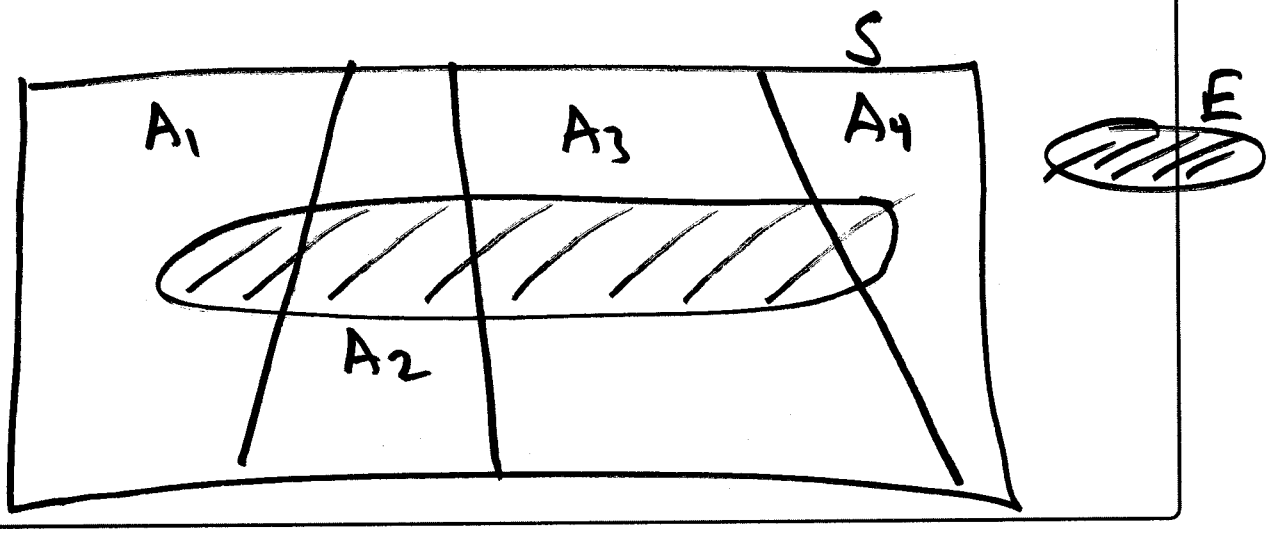
Theorem 1: Bayes' Theorem

1. Let S be a sample space partitioned into n events, A_1, A_2, ..., A_n.
2. Let E be an event in S

The probability of A_i, given E has occured $P(A_i|E) = \dfrac{P(A_i)P(E|A_i)}{P(E)}$

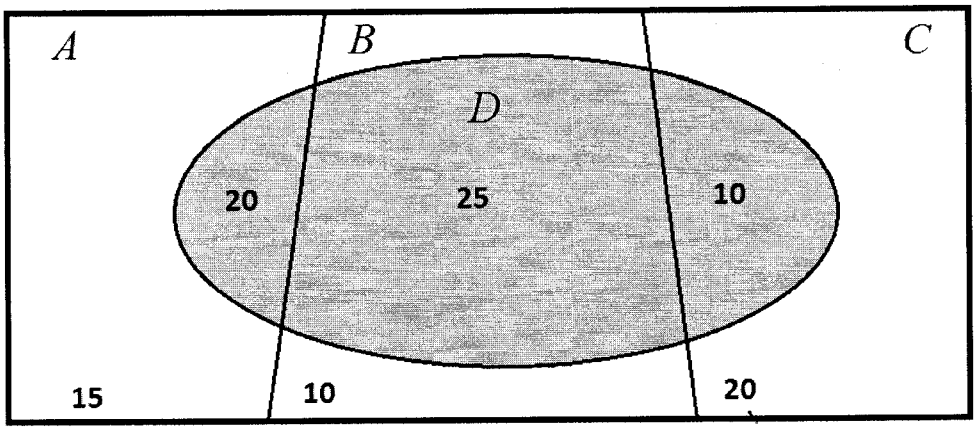
$$= \frac{P(A_i)P(E|A_i)}{P(A_1)P(E|A_1) + P(A_2)P(E|A_2) + P(A_3)P(E|A_3) + \cdots + P(A_n)P(E|A_n)}$$

MY WAY: $P(A_i|E) = \dfrac{P(A_i \cap E)}{P(E)}$



Example 1

Refer to the venn diagram. Use it to answer the following questions



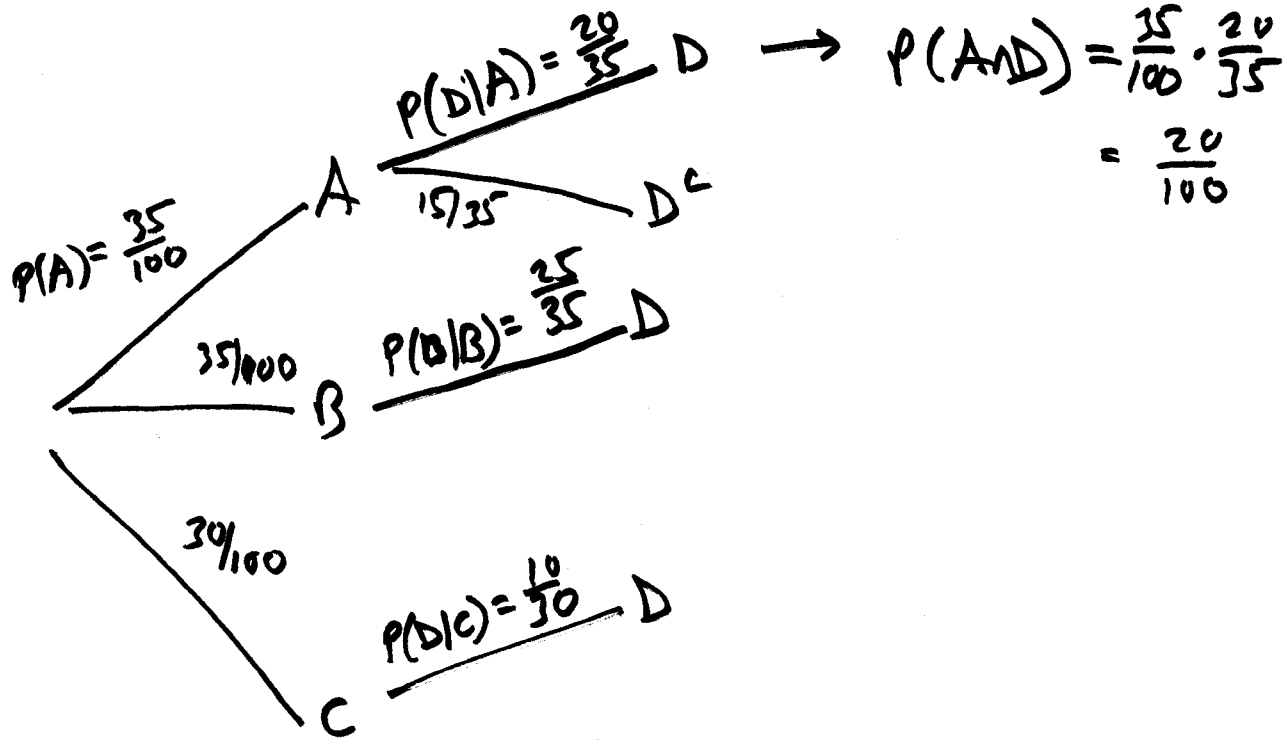
1. $P(D) = \frac{55}{100} = \frac{11}{20}$

2. $P(A|D) = \frac{20}{55} = \frac{4}{11}$

3. $P(D^c) = \frac{45}{100} = \frac{9}{20}$

4. $P(B|D^c) = \frac{10}{45} = \frac{2}{9}$

Draw a tree diagram



$P(A \cap D) = \frac{35}{100} \cdot \frac{20}{35} = \frac{20}{100}$

$P(D) = P(A \cap D) + P(B \cap D) + P(C \cap D)$

$P(B \cap D) = P(D|B)P(B)$

Example 3: Use class survey from 7.5

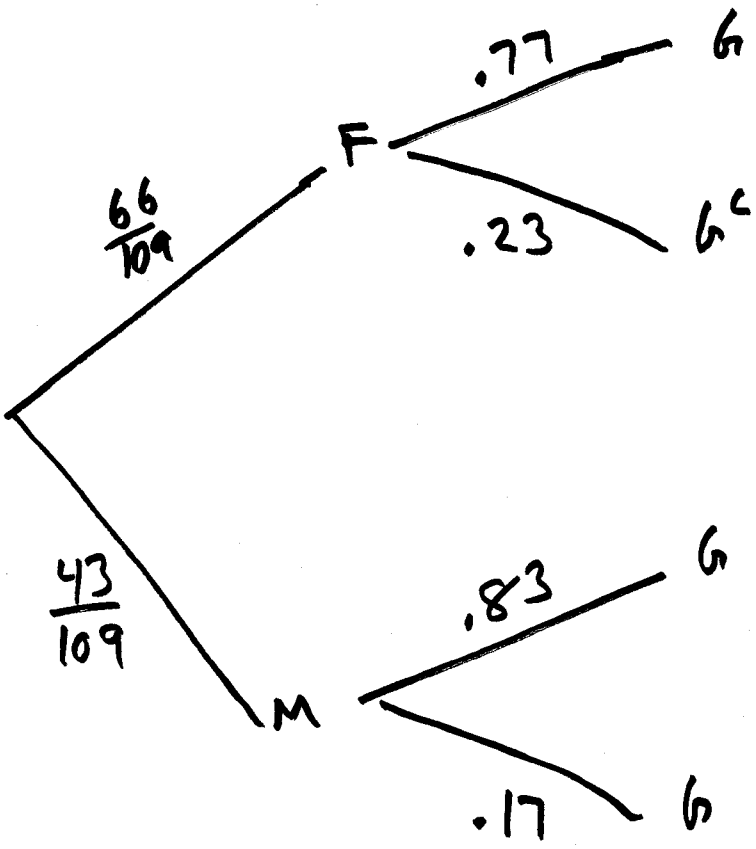
In a survey of 109 students, it was found that 66 were female. Additionally, it was found that 77 % of those females play games, whereas 83 % of the males play games. Find the probability a randomly selected student

1. is female given they play games?

$P(F|G)$

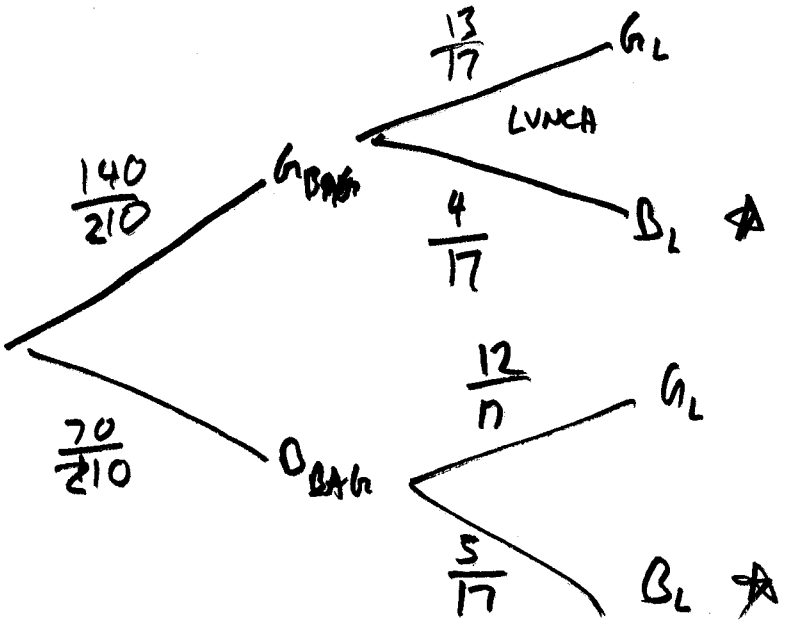
$P(G|F) = 0.77$

$P(G|M) = .83$



Example 2

210 M&Ms are in a bag (140 green, 70 blue). I currently have in my lunchbox 12 green and 4 blue. A M&M is drawn at random from the bag and placed in my lunch box. During lunch I randomly select a blue M&M. What is the probability the transferred M&M from the bag was green?



$$P(G_{BAG} \mid B_L) = \frac{P(G_{BAG} \cap B_L)}{P(B_L)} = \frac{\frac{140}{210} \times \frac{4}{17}}{\frac{140}{210} \times \frac{4}{17} + \frac{70}{210} \times \frac{5}{17}}$$

$$= \frac{8/51}{\frac{8}{51} + \frac{5}{51}}$$

$$= \frac{8}{13} \text{ OR}$$

$$= .61$$