MATH 210 FINITE MATHEMATICS

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7.5 Conditional Probability and Independence

Example 1

A survey was done of students playing video games / board games.

	play games (G)	does not play games (G^c)	Total
Female (F)	51	15	66
Male (F^c)	36	7	43
Total	87	22	109

A student was selected randomly.

1. What is the probability that the student plays game? $\frac{87}{109} \approx .80 = P(G)$

2. What is the probability that the student does not play games?

$$P(G^c) = 1 - P(G) = 1 - .8 = .2 \quad \text{or } \frac{22}{109}$$

3. What is the probability that the student is a female that plays game?

$$P(F \cap G) = \frac{51}{109} = .47$$

4. What is the probability that the student is a male that plays game?

$$P(M \cap G) = \frac{36}{109} \approx .33$$

5. Given the student is female, what is the probability that she plays games?

$$P(G|F) = \frac{51}{66} = .77 \qquad \frac{P(G \cap F)}{P(F)} = \frac{51/\cancel{109}}{66/\cancel{109}}$$

6. Given the student plays game, what is the probability the student is female?

$$P(F|G) = \frac{51}{87} = .59$$

Definition 1: Conditional Probability

The probability that event B occurs given that event A has already occurred is given by

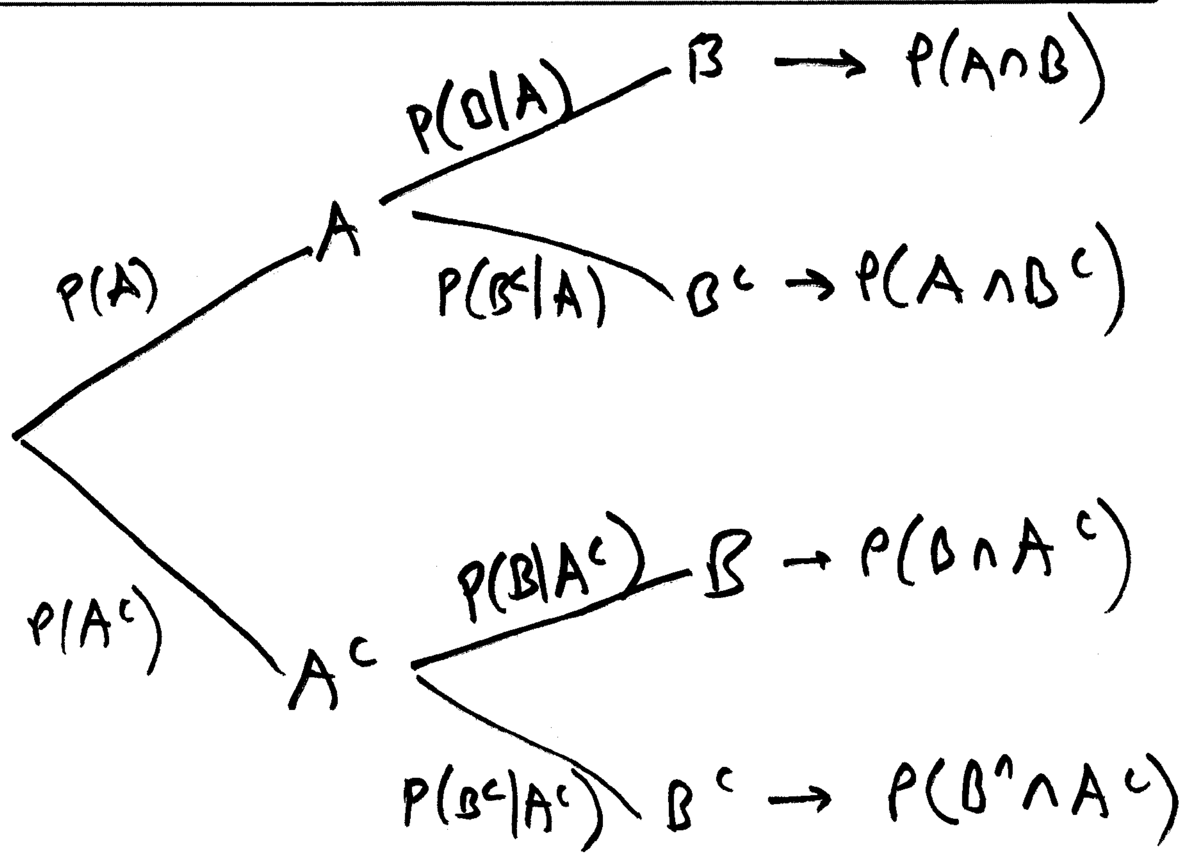
$$\underbrace{P(B \mid A)} = \frac{P(A \cap B)}{\underbrace{P(A)}}$$

which can be rewritten as

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

Example 2

Consider the tree diagram to help understand the formula



Definition 2: Independent Events

If A and B are independent events, then

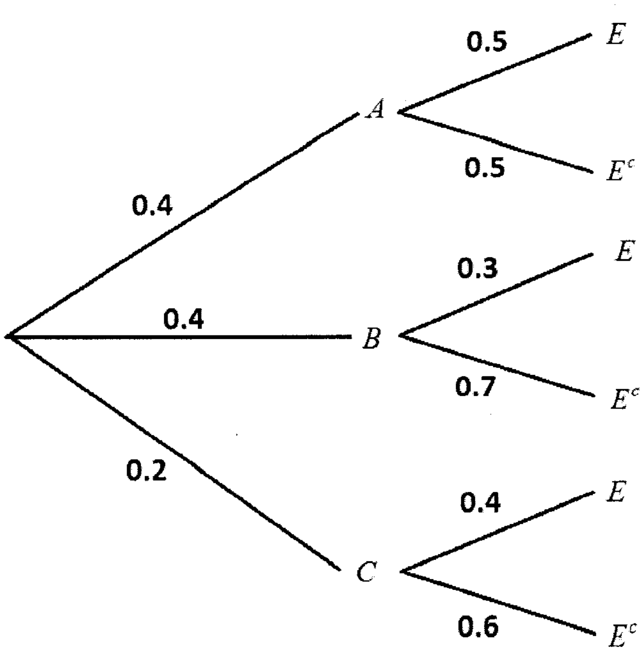
$$P(A|B) = P(A)$$

$$P(B|A) = P(B)$$

$$[P(A \cap B) = P(A) \times P(B)]$$

Example 3

Consider the tree diagram. Find the following

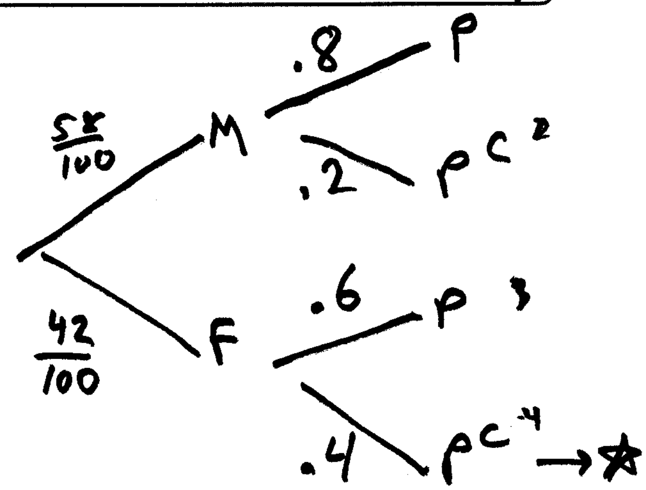


$A \cap E$

1. $P(A) = 0.4$
2. $P(E|A) = 0.5$
3. $P(A \cap E) = P(E|A) \cdot P(A)$
   $= 0.5 \times 0.4$
   $= 0.2$
4. $P(E) = P(A \cap E) + P(B \cap E) + P(C \cap E)$
   $= .4 \cdot .5 + .4 \cdot .3 + .2 \cdot .4$
   $= .4$
5. Does $P(A \cap E) = P(A) \cdot P(E)$?
   $.2 = .4 \cdot .4$
   $.2 = .16$? NO
6. Are A and E independent?
   NO

Example 4

There 100 students in this class, of which 58 are male and 42 are female. It is known that 80% of the males and 60% of the females buy popcorn at the movies. If a student is selected at random, what is the probability that the student



1. is a female who does not buy popcorn?

$$P(F \cap P^c)$$
$$= \frac{42}{100} \cdot 0.4 = .168$$

2. is a male that buys popcorn?

$$P(M \cap P) = \frac{58}{100} \cdot 0.8$$
$$= .464$$

$$P(P|M) = .8$$

3. is a female that buys popcorn?

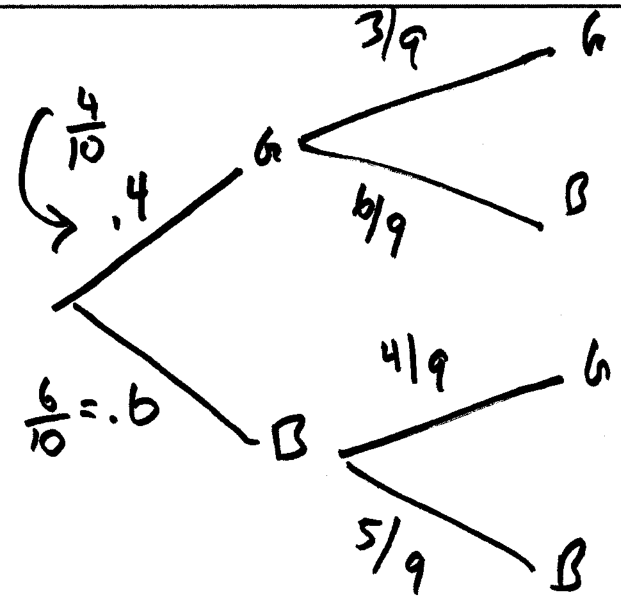
$$P(F \cap P) = \frac{42}{100} \cdot 0.6 = .252$$

4. buys popcorn? $P(\text{POPCORN}) = P(M \cap P) + P(F \cap P)$

$$= .464 + .252$$
$$= .716$$

Example 5

10 M&Ms are placed in a bag (4 green, 6 blue). Two M&Ms are drawn in succession. What is the probability the second M&M is green if



1. the first M&M is blue?

$$P(G_2 \mid B_1) = \frac{4}{9} = .44\ldots$$

2. ~~the second M&M is drawn without replacing the first?~~

WHAT IF $P(G_2 \mid B) = P(G)$?

DID CHOOSING B FIRST MATTER?

SO B AND G ARE INDEPENDENT

3. ~~the first M&M is replaced before the second is drawn?~~