

MATH 210 FINITE MATHEMATICS

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7.4 Counting Techniques in Probability

Definition 1

Let S be a uniform sample space and let E be any event. Then,

$$P(E) = \frac{n(E)}{n(S)} \quad n(E) \text{ IS \# OF OUTCOMES IN } E$$

Example 1

An unbiased coin is tossed six times. What is the probability that the coin will land heads

1. Exactly three times?

6 TOSSES, 3 HEADS

$$P(3 \text{ HEADS}) = \frac{C(6,3)}{2^6} = \frac{20}{64} = \frac{5}{16} = .3125$$

2. At least three times?

$$3: \frac{C(6,3)}{64}$$

$$4: \frac{C(6,4)}{64}$$

$$5: \frac{C(6,5)}{64}$$

$$6: \frac{C(6,6)}{64}$$

ADD TOGETHER

$$P(\text{AT LEAST } 3)$$

$$= \frac{42}{64} = \frac{21}{32}$$

Example 2

You're playing with a standard deck of cards. Find the probability that

1. one card is selected and it is a king?

$$P(\text{KING}) = \frac{4}{52} = \frac{1}{13}$$

2. two kings are selected? ~~0.004~~

• HAVE 4 KINGS, WANT 2: $C(4,2)$

• TOTAL OUTCOMES, HAVE 52 CARDS, WANT 2: $C(52,2)$

$$P(2 \text{ KINGS}) = \frac{C(4,2)}{C(52,2)} = \frac{6}{1326} \approx .004 \text{ OR } 0.4\%$$

3. the five cards are selected and contain 3 kings and 2 10s?

KINGS: ~~4~~ HAVE 4, WANT 3 $C(4,3)$

10s: HAVE 4, WANT 2 $C(4,2)$

TOTAL OUTCOMES

HAVE 52, WANT 5 $C(52,5)$

$$\begin{aligned} P(3K \text{ AND } 2 \text{ 10s}) &= \frac{C(4,3) \times C(4,2)}{C(52,5)} \\ &= \frac{24}{2598960} = .000009234 \\ &= .00092\% \end{aligned}$$

Example 3

An exam consists of ⁵ multiple choice questions each having 5 choices (only one is correct). If a student randomly guesses on each question,

1. what is the probability he or she will get exactly ⁴ correct?

C W C C C

OF WAYS OF ANSWERING IS 5^5

OF WAYS OF GETTING 4 CORRECT $C(5,4)$

OF WAYS OF GETTING 1 WRONG 4^1

2. what is the probability you get at least 3 right?

$$P(4 \text{ CORRECT}) = \frac{C(5,4) \times 4^1}{5^5}$$

$$3 \text{ RIGHT} \quad \frac{C(5,3) \times 4^2}{5^5} = .0512 = .0064$$

$$4 \text{ RIGHT} : .0064$$

$$5 \text{ RIGHT} : \frac{C(5,5) \times 4^0}{5^5} = .00032$$

$$\text{TOTAL: } .0608 \text{ OR } 6\%$$

Example 4

You have a jar with 5 green Skittles, 3 red, and 7 blue. If you select 4 skittles at random, what is the probability that you select

1. exactly one red and three blue? RED: HAVE 3, WANT 1 $C(3,1)$
 BLUE: HAVE 7, WANT 3 $C(7,3)$
 GREEN: HAVE 5, WANT 0 $C(5,0)$

$$P(1R, 3B) = \frac{C(3,1) \times C(7,3) \times C(5,0)}{C(15,4)} = \frac{105}{1365} = .077 \text{ or } 7.7\%$$

2. all green?

$$\frac{C(3,0) \times C(7,0) \times C(5,4)}{C(15,4)} \text{ OR } \frac{C(5,4)}{C(15,4)} = \frac{5}{1365}$$

$$= .0037$$

$$= \text{.37\%}$$

Example 5

Two cards are selected. Find the probability that they have the same rank.

$$13 \times C(4,2) \text{ OR } C(13,1) \times C(4,2)$$

$$P(\text{PAIR}) = \frac{13 \times C(4,2)}{C(52,2)} = \frac{78}{1326} = \frac{1}{17} = .059$$

$$\approx 6\%$$