

MATH 210 FINITE MATHEMATICS

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7.3 Rules of Probability

Definition 1: Properties

1. ~~$P(E)$~~ $0 \leq P(E) \leq 1$
2. $P(S) = P(\text{SAMPLE SPACE}) = 1$
3. IF E AND F ARE MUTUALLY EXCLUSIVE
4. $P(E \cup F) = P(E) + P(F)$
- ~~IF NOT~~ $P(E \cup F) = P(E) + P(F) - P(E \cap F)$
5. $P(E^c) = 1 - P(E)$

Example 1

Let E and F be two events of an experiment. Suppose $P(E) = 0.6$, $P(F) = 0.5$, and $P(E \cup F) = 0.85$. Find

$$1. P(F^c) = 1 - P(F) = 1 - 0.5 = 0.5$$

$$2. P(E^c) = \cancel{1 - P(E)} = 1 - P(E) = 1 - 0.6 = 0.4$$

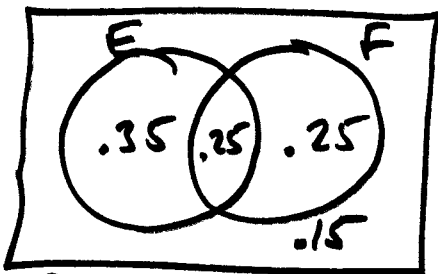
$$3. P(E \cap F) \text{ USE } P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$0.85 = 0.6 + 0.5 - P(E \cap F)$$

$$0.85 = 1.1 - P(E \cap F)$$

$$0.25 = P(E \cap F)$$

4. $P(E^c \cap F^c)$



$P(E^c \cap F^c) = .15$

OTHER WAY

NOTE: $E^c \cap F^c = (E \cup F)^c$

$$P((E \cup F)^c) = 1 - P(E \cup F)$$

$$= 1 - .85$$

$$= .15$$

5. $P(E \cup F^c)$

USE VENN

$$\rightarrow .35 + .25 + .15$$

$$= .75$$

Example 2

A card is drawn from a well-shuffled standard deck of 52 cards.

1. What is the probability it is not a 5?

$$\frac{48}{52} = \frac{12}{13} \text{ OR } 1 - P(5)$$

2. What is the probability that is a king or a heart?

$$P(K \cup H) = P(K) + P(H) - P(K \cap H)$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13} \approx .31$$

3. What is the probability that it is a spade or a heart?

$$P(S \cup H) = P(S) + P(H) - P(S \cap H)$$

$$= \frac{13}{52} + \frac{13}{52} - \frac{0}{52}$$

$$= \frac{26}{52} = \frac{1}{2} = 0.5$$

4. What is the probability it is a face card or a red?

$$\begin{aligned}
 P(F \cup R) &= P(F) + P(R) - P(F \cap R) \\
 &= \frac{12}{52} + \frac{26}{52} - \frac{6}{52} \\
 &= \frac{32}{52} = \frac{8}{13} \approx .62
 \end{aligned}$$

Example 3

1089 adults were asked "How serious a threat is climate change?" The results are:

Answer	Very Serious	Serious	Somewhat Serious	Not at all
Responses (%)	38	46	15	1

What is the probability that a person chosen at random

1. answered with a serious threat?

$$P(\text{SERIOUS THREAT}) = .46$$

2. answered with a very serious threat or a somewhat serious threat?

$$\begin{aligned}
 P(VS \cup SS) &= P(VS) + P(SS) - P(VS \cap SS) \\
 &= .38 + .15 - 0 = .53
 \end{aligned}$$

3. showed some concern?

$$.38 + .46 + .15 = .99$$