MATH 210 FINITE MATHEMATICS

BRIAN VEITCH • FALL 2016 • NORTHERN ILLINOIS UNIVERSITY

7.3 Rules of Probability

Definition 1: Properties

1. $0 \le P(E) \le 1$
2. $P(S) = 1$ S = SAMPLE OUTCOME
3. IF E AND F ARE MUTUALLY EXCLUSIVE
4. THEN $P(E \cup F) = P(E) + P(F)$

   IF NOT, $P(E \cup F) = P(E) + P(F) - P(E \cap F)$
5. $P(E^c) = 1 - P(E)$

Example 1

Let E and F be two events of an experiment. Suppose $P(E) = 0.6$, $P(F) = 0.5$, and $P(E \cup F) = 0.85$. Find

1. $P(F^c)$

$$P(F^c) = 1 - P(F) = 1 - .5 = .5$$

2. $P(E^c)$

$$1 - P(E) = 1 - .6 = .4$$

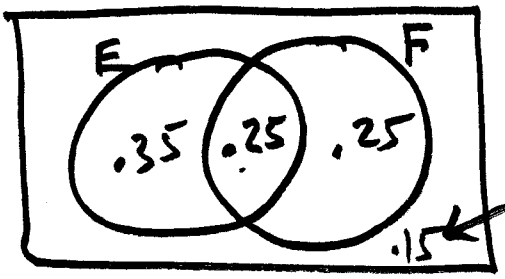
3. $P(E \cap F)$

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$
$$.85 = .6 + .5 - P(E \cap F)$$
$$.85 = 1.1 - P(E \cap F)$$
$$-.25 = -P(E \cap F)$$
$$0.25 = P(E \cap F)$$

$P(E^c \cup F^c) = P(E^c) + P(F^c) - P(E^c \cap F^c)$

4. $P(E^c \cap F^c)$

$P(E \cap F) = .25 \quad P(E) = .6$



$P(E^c \cap F^c) = .15$

5. $P(E \cup F^c)$

$P(E) + P(F^c) - P(E \cap F^c)$

$= .6 + .5 - .35$

$= .75$

Example 2

A card is drawn from a well-shuffled standard deck of 52 cards.

1. What is the probability it is not a 5? $P(5) = \frac{4}{52}$

so $P(\text{NO } 5) = 1 - P(5) = 1 - \frac{4}{52} = \frac{48}{52}$

2. What is the probability that is a king or a heart?

$P(K \cup H) = P(K) + P(H) - P(K \cap H) \approx .92$

$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$

$\approx .31$

3. What is the probability that it is a spade or a heart?

$P(S \cup H) = P(S) + P(H) - P(S \cap H)$

$= \frac{13}{52} + \frac{13}{52} - 0 = \frac{1}{2}$

4. What is the probability it is a face card or a red?

$$P(F \cup R) = P(F) + P(R) - P(F \cap R)$$
$$= \frac{12}{52} + \frac{26}{52} - \frac{6}{52}$$
$$= \frac{32}{52} = \frac{8}{13} \approx .615$$

Example 3

1089 adults were asked "How series a threat is climate change?" The results are:

Answer	Very Serious	Serious	Somewhat Serious	Not at all
Responses (%)	38	46	15	1

What is the probability that a person chosen at random

1. answered with a serious threat? $P(\text{SERIOUS}) = .46$

2. answered with a very serious threat or a somewhat series threat?

$$P(VR \cup SS) = .38 + .15 - 0$$
$$= .53$$

3. showed some concern?

$$P(\text{SOME CONCERN}) = 1 - P(\text{NO CONCERN})$$
$$= 1 - .01$$
$$= .99$$

Example 4

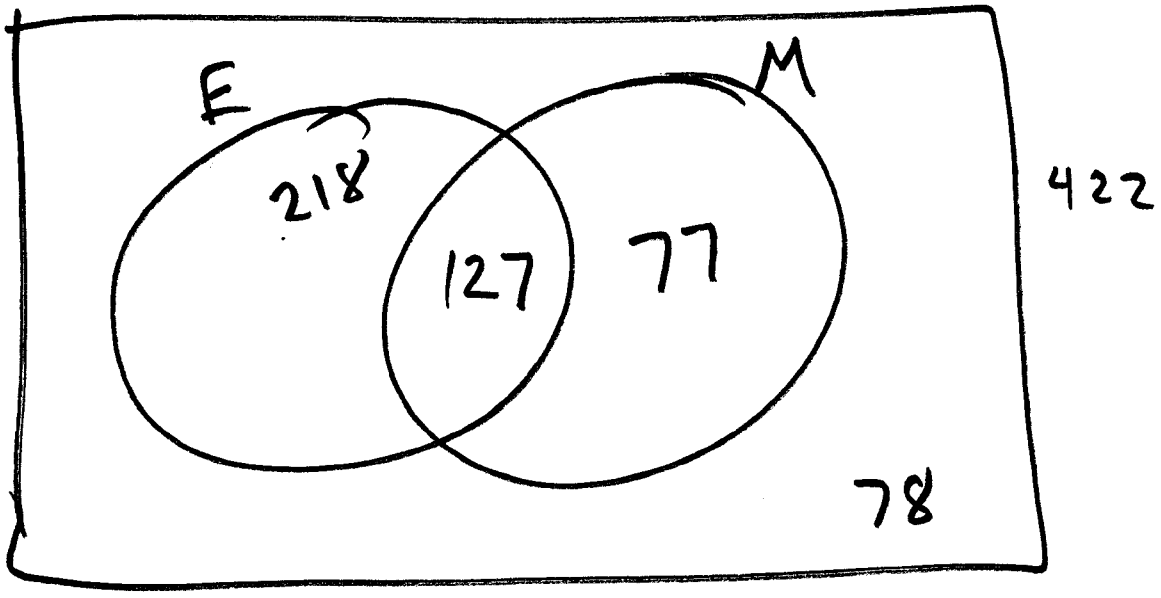
A pair of dice is rolled and the number that appears upper most on each die is observed.

1. What is the probability a double is NOT thrown?

2. Find the probability the sum is at least 7.

Example 5

Among 500 freshmen pursuing a business degree at a university, 345 are enrolled in an economics course, 204 are enrolled in a mathematics course, and 127 are enrolled in both an economics and a mathematics course. What is the probability that a freshman selected at random from this group is enrolled in each of the following?



1. an economics and/or a mathematics course

$$P(E \cup M) = \frac{422}{500} = .844$$

2. exactly one of these two courses.

$$P(E \text{ ONLY OR } M \text{ ONLY}) = \frac{218}{500} + \frac{77}{500} = \frac{295}{500} = .59$$

3. neither an economics course nor a mathematics course.

$$P(E^c \cap M^c) = \frac{78}{500} = .156$$