

# MATH 210 FINITE MATHEMATICS

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## 7.2 Probability

### Definition 1: Relative Frequency

Suppose we repeat an experiment  $n$  times and an event  $E$  occurs  $m$  times. The relative frequency of  $E$  is

$$\frac{m}{n} \leftarrow \begin{array}{l} E \text{ OCCURS} \\ \text{TOTAL OUTCOMES} \end{array}$$

### Definition 2: Simple Event

EVENT WITH ONLY ONE OUTCOME

### Example 1

NIU was surveyed and the following info was found:

	Undecided	Liberal Arts	Health	Business	Total
Upper-classmen	12	46	24	88	170
Lower-classmen	60	15	7	8	110
Total	72	61	31	116	280

If one NIU student is selected at random, what is the probability that

- an upper-classmen is selected?  $= \frac{170}{280} = \frac{17}{28} \approx .607$   
60.7%
- an undecided lower-classmen is selected?  $= \frac{60}{280} = \frac{3}{14} \approx .214$   
OR 21.4%
- a liberal arts student is selected?  $= \frac{61}{280} \approx .218$

**Definition 3: Probability Distribution**

A table that lists the probability of each event

Probability Distribution		
	Event	Probability
<p><b>EVENTS ARE MUTUALLY EXCLUSIVE</b></p> <p>1. Probability Function <math>P(E_i)</math></p> <p>2. <math>P(E_1) + P(E_2) + \dots + P(E_n) = 1</math></p> <p>3. <math>P(E_1 \cup E_2) = P(E_1) + P(E_2)</math></p> <p>4. <math>0 \leq P(E_i) \leq 1</math></p>	$E_1$	$P(E_1)$
	$E_2$	$P(E_2)$
	$\vdots$	
	$E_n$	$P(E_n)$

**Definition 4: Uniform Sample Space**

**EAH EVENT HAS THE SAME PROBABILITY**

**Example 2: Rolling One Die**

Suppose you roll one die and record the number.

1. What is the sample space?

$\{1, 2, 3, 4, 5, 6\}$

2. List all simple events

$\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}$   
 UNIFORM SAMPLE SPACE

3. What is the probability of each simple event?

$$P(\{1\}) = 1/6 \quad \text{SAME FOR } 3, 4, 5, 6$$

$$P(\{2\}) = 1/6$$

4. Find the probability distribution

Probability Distribution	
Simple Event	Probability
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

5. What is the probability of rolling an odd?  $P(1 \text{ OR } 3 \text{ OR } 5) = \frac{3}{6}$

$$= P(1) + P(3) + P(5)$$

$$= 1/6 + 1/6 + 1/6 = 3/6 \quad \leftarrow \frac{1}{2}$$

**Example 3**

Create a probability distribution for the sum of two dice.

SUM	PROB	SUM	PROB
2	$\frac{1}{36}$	10	$\frac{3}{36}$
3	$\frac{2}{36}$	11	$\frac{2}{36}$
4	$\frac{3}{36} \approx .083$	12	$\frac{1}{36}$
5	$\frac{4}{36}$		
6	$\frac{5}{36}$		
7	$\frac{6}{36}$		
8	$\frac{5}{36}$		
9	$\frac{4}{36}$		

**Example 4**

Using empirical data create a probability distribution for 3 dice.

SUM	PROB	SUM	PROB
3	.004	12	.11363
4	.0129	13	
5	.02707	14	
6	.04613	15	
7	.07058	16	
8		17	
9	.11735	18	
10	.12441		
11	.12985		

$$P(3) = \frac{1}{216} \approx .0046$$

**Example 5**

A group of people were asked to name their favorite class.

Class	Math	English	Sociology	Music	Economics
Frequency	7	4	10	9	7

1. Find the probability distribution

CLASS	MATH	E	S	M	ECOW
PROB	$\frac{7}{37}$	$\frac{4}{37}$	$\frac{10}{37}$	$\frac{9}{37}$	$\frac{7}{37}$

$\uparrow$        $\uparrow$        $\uparrow$   
 .189    .108    .27

2. Is this a uniform sample space?

No

3. What is the probability that a student selected randomly has English or Music as their favorite class?

$$\begin{aligned}
 & P(\text{ENGLISH OR MUSIC}) \\
 &= P(\text{ENGLISH}) + P(\text{MUSIC}) \\
 &= \frac{4}{37} + \frac{9}{37} = \frac{13}{37}
 \end{aligned}$$