MATH 210 FINITE MATHEMATICS

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6.4 Permutations and Combinations

Definition 1: n-Factorial

For any natural number n $n! = n\cdot(n-1)(n-2)(n-3)\cdots 3\cdot 2\cdot 1$

$7! = 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1 = 50$

NOTE: $1! = 1$, $0! = 1$

Definition 2: Permutations

SET OF OBJECTS WHERE ARRANGEMENT IS IN A SPECIFIC ORDER

YOU HAVE n OBJECTS, CHOOSE r OF THEM

$$P(n,r) = \frac{n!}{(n-r)!}$$

Example 1: Permutations

Eight horses are entered in a race. How many ways first, second, and third place results are there?

- 8 OBJECTS, CHOOSE 3

$$P(8,3) = \frac{n!}{(n-r)!} = \frac{8!}{(8-3)!} = \frac{8!}{5!} = 336$$

$$= \frac{8\cdot 7\cdot 6\cdot \cancel{5\cdot 4\cdot 3\cdot 2\cdot 1}}{\cancel{5\cdot 4\cdot 3\cdot 2\cdot 1}} = 8\cdot 7\cdot 6$$

$$= 336$$

Example 2: Permutations

Jack, Jill, and five of their friends go to the movies. They all sit next to each other in the same row. How many ways can this be done?

- 7 OBJECTS
- CHOOSE 7 (ALL OF THEM)

\# OF WAYS $P(7,7) = \frac{7!}{(7-7)!} = \frac{7!}{0!}$

$= 7! = 5040$

Example 3: Permutations

Find the number of ways a president, vice president, secretary, and treasurer can be chosen from a committee of ten members?

TASKS $\underset{P}{\underline{10}} \times \underset{VP}{\underline{9}} \times \underset{S}{\underline{8}} \times \underset{T}{\underline{7}}$

$P(10,4) = \frac{10!}{(10-4)!} = \frac{10!}{6!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6!}}{\cancel{6!}}$

$= 10 \cdot 9 \cdot 8 \cdot 7$

$= 5040$

Definition 3: Combinations

- n DISTINCT OBJECTS, WANT r OF THEM
- PLACEMENT ORDER DOES NOT MATTER

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

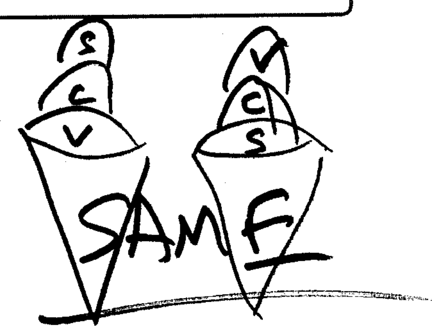
EX) $C(9,3) = \frac{9!}{3!(9-3)!} = \frac{9!}{3!6!} = 84$

Example 4: Combinations

Suppose you order three scoops of ice cream at Baskin-Robbins? Assuming you want three different types of ice cream, how many different ways can you do this?

ORDER DOES NOT MATTER

- 31 FLAVORS TO CHOOSE FROM



- CHOOSE 3
- \# OF WAYS IS $C(31,3)$

$= 4,495$ DIFFERENT 3 SCOOP COMBOS

If you were to buy one of these combinations a day, how long would it take to eat each one?

Example 5

You are play a trading card game that has 30 cards. You have 25 regular and 5 rare cards.

1. If you randomly select 8 cards, how many samples have two rare cards?

25 REGULAR CARDS - CHOOSE 6: C(25,6)
5 RARE CARDS - CHOOSE 2 C(5,2)

OF WAYS IS C(25,6) × C(5,2)
177,100 × 10
= 1,771,000 WAYS

2. How many will have at least 7 regular cards? 7 REG OR 8 REG

7 REGULAR
1 RARE

$\underbrace{C(25,7)}_{\text{REGULAR}} \times \underbrace{C(5,1)}_{\text{RARE}}$

(480,700) × (5) = 2,403,500

8 REGULAR
0 RARE

C(25,8) × C(5,0)
= 1081575 × 1
= 1,081,575

ADD 3,485,075 WAYS

3. How many will have at least 1 rare card?

TWO OPTIONS: (1) 1 RARE OR 2 RARE ... OR 5 RARE
OR
(2) TOTAL POSSIBILITIES − 0 RARE CARDS

C(30,8)

C(25,8) × C(5,0)

= 5,852,925 − 1,081,575
= 4,771,350

Example 6

Suppose we have a bag of M&Ms containing 6 blue, 3 red, and 7 green. You choose 5 at random.

1. How many samples of 5 pieces can be chosen? Do the colors matter?

NO

$$c(16,5) = 4368$$

2. How many samples are there in which all are green?

BLUE: $C(6,0)$, RED: $C(3,0)$, GREEN: $C(7,5)$

$$C(6,0) \times C(3,0) \times C(7,5)$$

$$1 \times 1 \times 21$$

$$= 21 \text{ SAMPLES}$$

3. How many samples are there in which 2 are blue and 1 is red?

$$\underbrace{C(6,2)}_{\text{BLUE}} \times \underbrace{C(3,1)}_{\text{RED}} \times \underbrace{C(7,2)}_{\text{GREEN}}$$

$$15 \times 3 \times 21$$

$$= 945$$