Math 210 Finite Mathematics

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5.2 Annuities

Definition 1: Annuity

A SEQUENCE OF PAYMENTS

Definition 2: Future Value of an Annuity

The future value S of an annuity of n payments of R dollars earning interest rate of i per period is

$$S = R\left[\frac{(1+i)^n - 1}{i}\right] \qquad i = \frac{r}{m}$$

$$n = mt$$

Example 1

John will need \$12,000 for a down payment in three years. He deposits \$200 per month earning 5% per year compounded monthly. Will he have enough?

$$R = \$200, \quad r = .05, \quad m = 12, \quad i = \frac{.05}{12}$$

$$S = 200 \cdot \left[\frac{\left(1 + \frac{.05}{12}\right)^{12 \cdot 3} - 1}{.05/12}\right] = \frac{.1614722313}{.00416667}$$

$$= 200\,(38.753305)$$

$$= \$7750.66$$

NOT ENOUGH

YOU SPENT
$200 \times 36 = \$7200$

INTEREST EARNED
$7750.66 - 7200 = \$550.66$

Example 2

Suppose John consulted you on what he would have to save per month so he has \$12,000 in three years. At an interest rate of 5% per year compounded monthly, what would he save to deposit per month?

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$12000 = R\left[\frac{(1+.05/12)^{12\cdot 3} - 1}{.05/12}\right]$$

$$12000 = R(38.753305)$$

$$\frac{12000}{38.753305} = R$$

$$\boxed{\$309.65 = R}$$

Definition 3: Present Value of an Annuity

The present value of P of an annuity consisting of n payments of R dollars each, earning interest at i per period is

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

Example 3

Find the present value of an annuity consisting of 24 quarterly payments of \$250 each and earning 3% per year compounded quarterly.

$$P = 250\left[\frac{1-(1+.03/4)^{-24}}{.03/4}\right]$$

$$= 250\left[\frac{.1641686}{.0075}\right]$$

$$= \$5472.29$$

Example 4

Suppose you're 22 years old, just graduated from college, and begin thinking of retirement. There are many options out there. Which option is better?

1. You get a job and deposit \$150 per month into an account earning 5% per year compounded monthly for 7 years. You leave this money alone until the age of 65.

2. It's too hard to save when you're 22. You wait until you're 45 years old and then deposit \$150 per month at 5% per year compounded monthly. How much will you have when you're 65 years old?

(1) AFTER 7 YEARS: $S = 150\left[\frac{(1+.05/12)^{12\cdot 7}-1}{.05/12}\right] = \$15,049.30$

LET IT SIT FOR 36 YEARS → USE $P(1+i)^n$

$$15049.30(1+.05/12)^{12\cdot 36} = \$90,703.13$$

\$ SPENT $150 \times 12 \times 7 = \$12,600$

(2) $$\$ S = 150\left[\frac{(1+.05/12)^{12\cdot 20}-1}{.05/12}\right] = \$61,655.05$$

SPENT $\$150 \times 12 \times 20 = \$36000$

Example 5

Brian payed a down payment of \$12,000 towards a new car. He secured a loan for 60 months at an interest rate of 1.99% per year compounded monthly. His monthly payments are \$232 per month. How much was the car worth?

CAR PRICE = PRESENT VALUE OF LOAN + DOWN PAYMENT

\$12000

[redacted] $n=60$

$r=.0199$

$m=12$

$R=\$232$

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right] \qquad i=\frac{r}{m}$$

$$= 232\left[\frac{1-\left(1+\frac{.0199}{12}\right)^{-12\cdot 5}}{\left(.0199/12\right)}\right]$$

$$= 232\left[\frac{.09463544}{.00165833}\right]$$

$$= 232\left[57.0667117\right]$$

LOAN AMOUNT = 13,239.48

CAR PRICE = 13,239.48 + 12,000 = \$25,239.48