MATH 210 FINITE MATHEMATICS

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5.1 Compound Interest

Definition 1: Principle

INITIAL AMOUNT

Definition 2: Simple Interest

YEARLY CHARGE, ONLY ON ORIGINAL PRINCIPLE

Definition 3: Simple Interest Formulas

I = INTEREST, P = PRINCIPLE, t = TIME,

r = ANNUAL INTEREST RATE

$I = Prt$

$A = P(1+rt)$ A FUTURE BALANCE

Example 1

A bank pays simple interest at the rate of 8% per year. If a customer deposits \$1000 and makes no withdrawals for 3 years, what is the total amount at the end of 3 years? What is the interest earned?

$I = Prt = 1000(.08)(3) = \$240$

TOTAL AMOUNT AFTER 3 YEARS

IS $1000 + 240 = \$1240$

Example 2

Suppose you buy at TV for \$1500 that is advertised at \$30 a month for 5 years. How much did you pay in interest?

YOU PAYED $30 \times 60 = \$1800$

BUT TV COSTS \$1500, SO INTEREST EARNED IS \$300

Example 3: Compound Interest

You put \$5000 in an account that pays 4% compound interest per year.

1. At the end of the 1st year: $P(1+rt)$

$A = 5000(1+.04 \cdot 1) = 5000(1+.04) = 5200$

EARNED \$200

2. At the end of the 2nd year:

$A = 5200(1+.04 \cdot 1) = 5200(1+.04) = 5408$

EARNED \$208

3. At the the end of the 3rd year:

$A = 5408(1+.04) = \$5624.32$

EARNED 216.32

Definition 4: Compound Interest Formula

$$A = P(1+i)^{\wedge}, \quad i = \frac{r}{m}, \quad n = mt$$

A = ACCUMULATED (FUTURE) AMOUNT
P = PRINCIPLE
r = NOMINAL INTEREST RATE (GIVEN)
m = # OF COMPOUNDING PERIODS PER YEAR
t = # OF YEARS

Example 4

If \$3000 is saved with an interest rate of 2% pear year. Find the accumulated amount after 3 years if compounded

1. annually $m=1$
2. semiannually $m=2$
3. quarterly $m=4$
4. monthly $m=12$
5. weekly $m=52$

$P = 3000$
$r = .02$
$t = 3$ YEARS

(1) ANNUALLY: $A = 3000\left(1+\frac{.02}{1}\right)^{1\cdot 3} = \$3183.62$

(2) SEMI-ANNUALLY $A = 3000\left(1+\frac{.02}{2}\right)^{2\cdot 3} = \$3184.56$

(3) QUARTELY: $A = 3000\left(1+\frac{.02}{4}\right)^{4\cdot 3} = \$3185.03$

(4) MONTHLY $A = 3000\left(1 + \frac{.02}{12}\right)^{12 \cdot 3} = \$3185.35$

(5) WEEKLY: $A = 3000\left(1 + \frac{.02}{52}\right)^{52 \cdot 3} = \$3185.47$

Example 5

What is the balance on a principle amount of \$3500, saved at 3% compounded monthly after (a) 1 month, (b) 1 year, (c) 10 years, (d) 40 years

(a) 1 MONTH $A = 3500\left(1 + \frac{.03}{12}\right)^{12 \cdot 1/12} = \$3508.75$

(b) 1 YEAR: $A = 3500\left(1 + \frac{.03}{12}\right)^{12 \cdot 1} = \$3606.46$

(c) 10 YEARS $A = 3500\left(1 + \frac{.03}{12}\right)^{12 \cdot 10} = \$4722.74$

(d) 40 YEARS $A = 3500\left(1 + \frac{.03}{12}\right)^{12 \cdot 40} = \$11,603.02$

Example 6

You deposit \$2000. Suppose the stated interest rate (nominal) is 4% compounded monthly. If you were to receive interest ONCE, what would the interest rate have to be so that you earn the same amount?

MONTHLY: $A = 2000\left(1 + \frac{.04}{12}\right)^{12 \cdot 1} = \$2081.48$

ONCE: $\frac{2081.48}{2000} = \frac{2000(1+r)^1}{2000}$

$1.0407415 = 1 + r$

$.0407415 = r$

or $r = \underbrace{4.074\%}_{APY}$

Definition 5

APY – EFFECTIVE INTEREST RATE

$r_{eff} = \left(1 + \frac{r}{m}\right)^m - 1$

APY

Example 7

Find the effective rate of interest corresponding to a nominal rate of 4% per year compounded (a) annually, (b) semiannually, (c) quarterly, (d) daily

1. Annually $(1+.04/1)^1 - 1 = .04$ or 4%

2. Semiannually $(1+.04/2)^2 - 1 = .0404$ or 4.04%

3. Quarterly $(1+\frac{.04}{4})^4 - 1 = .0406$ or 4.06%

4. Daily $(1+.04/365)^{365} - 1 = .0408$ or 4.08%

Definition 6: Present Value

$$P = A(1+i)^{-n}$$

$i = \frac{r}{m}$

$n = mt$

Example 8

Suppose that you want to take a trip to Europe in 6 years and figure you need \$7500. To have that much set aside in 6 years, how much do you have to deposit today into a bank earning 5% compounded quarterly?

$$P = A(1+i)^{-n}$$

$$P = 7500\left(1+\frac{.05}{4}\right)^{-4\cdot 6}$$

$$= \$5,566.48$$