MATH 210 FINITE MATHEMATICS

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4.3 Non-standard Simplex Problems

Definition 1: RECAP

Standard Maximization

1. Objective function to be maximized
2. All variables are non-negative $x \geq 0, y \geq 0$
3. Each linear constraint has the form

$$ax + by \leq c$$

Standard Minimization

1. Objective function to be minimized (coefficients are non-negative)
2. All variables are non-negative
3. Each linear constraint has the form

$$ax + by \geq c$$

Maximize $P = x + 3y$

$$\begin{aligned} 5x + 4y &\leq 32 \\ -x + 2y &\leq 10 \\ x \geq 0, y &\geq 0 \end{aligned}$$

STANDARD

Minimize $P = 21x - 3y$

$$\begin{aligned} x + y &\leq 32 \\ -3x + 2y &\leq 20 \\ x + 3y &\geq 2 \\ x \geq 0, y &\geq 0 \end{aligned}$$

NON-STANDARD

Steps 1: Simplex Method for Non-Standard Problems

1. If neccessary, rewrite the problem as a max. Minimizing C is equivalent as maximizing $-C$
2. If necessary, rewrite all constraints using $\leq$ signs
3. Introduce slack variables and set up initial simplex table
4. Scan the column of constants for negative numbers
   (a) If there are no negatives, complete the table using the standard technique
   (b) If there are negatives, go to step 5
5. Do the following:
   (a) Pick any negative number in the row in which a negative number is in the column of constants
   (b) This is your pivot column
   (c) Compute positive ratios of constants over column entry. The smallest positive ratio is the pivot row
   (d) Pivot the table around the pivot element
   (e) Return to step 4

Example 2

Minimize $C = 2x - 3y$

$$\begin{aligned} x + y &\leq 5 \\ x + 3y &\geq 9 \\ -2x + y &\leq 2 \\ x \geq 0, y &\geq 0 \end{aligned}$$

Example 1

Minimize $C = 2x - 3y$

$$\begin{aligned} x + y &\leq 5 \\ x + 3y &\geq 9 \\ -2x + y &\leq 2 \\ x \geq 0, y &\geq 0 \end{aligned}$$

MIXTURE OF INEQUALITIES

Why is this non-standard?

NEW SYSTEM

MAXIMIZE $P = -2x + 3y$

$x + y \leq 5$

$-x - 3y \leq -9$

$-2x + y \leq 2$

$x \geq 0, y \geq 0$

SLACK:

$x + y + s_1 = 5$

$-x - 3y + s_2 = -9$

$-2x + y + s_3 = 2$

OBJECTIVE: $2x - 3y + P = 0$

TABLE

x	y	s_1	s_2	s_3	P	C	
1	1	1	0	0	0	5	$5/1 = 5$
-1	-3	0	1	0	0	-9	$-9/-3 = 3$
-2	1	0	0	1	0	2	$2/1 = 2$
2	-3	0	0	0	1	0	

PIVOT

$R_1 - R_3$
$R_2 + 3R_3$
$R_4 + 3R_3$

x	y	s_1	s_2	s_3	P	C
3	0	1	0	-1	0	3
-7	0	0	1	3	0	-3
-2	1	0	0	1	0	2
-4	0	0	0	3	1	6

$-\frac{1}{7} R_2$

x	y	s_1	s_2	s_3	P	C
3	0	1	0	-1	0	3
1	0	0	-1/7	-3/7	0	3/7
-2	1	0	0	1	0	2
-4	0	0	0	3	1	6

$R_1 - 3R_2$
$R_3 + 2R_2$
$R_4 + 4R_2$

x	y	s_1	s_2	s_3	P	C
0	0	1	3/7	2/7	0	12/7
1	0	0	-1/7	-3/7	0	3/7
0	1	0	-2/7	1/7	0	20/7
0	0	0	-4/7	9/7	1	54/7

$\frac{7}{3} R_1$

x	y	s_1	s_2	s_3	P	C
0	0	7/3	1	2/3	0	4
1	0	0	-1/7	-3/7	0	3/7
0	1	0	-2/7	1/7	0	20/7
0	0	0	-4/7	9/7	1	54/7

$R_2 + \frac{1}{7}R_1$

$R_3 + \frac{2}{7}R_1$

$R_4 + \frac{4}{7}R_1$

x	y	s_1	s_2	s_3	P	C
0	0	7/3	1	2/3	0	4
1	0	0	-1/7	-3/7	0	3/7
0	0	-4/7	0	1/7	0	2/7
0						

x	y	s_1	s_2	s_3	P	C
0	0	7/3	1	2/3	0	4
1	0	11/3	0	-11/3	0	1
0	1	2/3	0	1/3	0	4
0	0	4/3	0	5/3	1	10

$x=1,\ y=4,\ P=10$

$C=-10$

Example 3

Maximize $P = 8x + 3y$

$$\begin{aligned} 2x + y &\leq 8 \\ -x + y &\geq 2 \\ x \geq 0, y &\geq 0 \end{aligned}$$

← $x - y \leq -2$

SLACK VARIABLES

$2x + y + s_1 = 8$

~~$-x + y - s_2 = 2$~~

$x - y + s_2 = -2$

OBJECTIVE: $-8x - 3y + P = 0$

INITIAL TABLE

x	y	s_1	s_2	P	C
2	1	1	0	0	8
1	-1	0	1	0	-2
-8	-3	0	0	1	0

PIVOT

DO SIMPLEX METOD TO GET

x	y	s_1	s_2	P	C
1	0	1/3	1/3	0	2
0	1	1/3	-2/3	0	4
0	0	11/3	2/3	1	28

$x = 2$
$y = 4$
$P = 28$