

MATH 210 FINITE MATHEMATICS

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4.2 Linear Programming: Minimization / Dual Problems

Example 1

Suppose you're given the following linear programming problem

$$\text{Minimize } C = -2x - 3y$$

$$5x + 4y \leq 32$$

$$x + 2y \leq 10$$

$$x \geq 0, \quad y \geq 0$$

Definition 1: The Dual Problem for Standard Minimization Problems

1. The objective function is to be minimized
2. All variables are non-negative
3. All other constraints must have the form

$$ax + by \geq C$$

Steps 1: Setting up the Dual Table

1. Write the table of data from original problem
2. Put objective function at bottom (without minus signs)
3. Reverse the rows and columns (transpose)
4. Change all inequalities to \leq
5. Use s_1, s_2, \dots for standard variables
6. Use original variables as the slack variables

Example 2

$$\text{Minimize } C = 10x + 11y$$

$$20x + 10y \geq 300$$

$$15x + 15y \geq 300$$

$$10x + 20y \geq 250$$

$$x \geq 0, y \geq 0$$

Example 3

$$\text{Minimize } C = 40x + 30y + 11z$$

$$2x + y + z \geq 8$$

$$x + y - z \geq 6$$

$$x \geq 0, y \geq 0$$

Example 4

An oil company operates two refineries. Refinery I has an output of 200, 100, and 100 barrels of low, medium, and high grade oil per day. Refinery II has an output of 100, 200, and 600 barrels of low, medium, and high grade oil per day. The company wishes to produce at least 1000, 1500, and 3000 of low, medium, and high grade oil to fill an order. If it costs \$200/day to operate Refinery I and \$300/day to operate Refinery II, determine how many days each refinery should be operated to meet the production requirements at minimum cost to the company? What is the minimum cost?