MATH 210 FINITE MATHEMATICS

BRIAN VEITCH • FALL 2016 • NORTHERN ILLINOIS UNIVERSITY

4.1 Linear Programming: The Simplex Method

Definition 1: Standard Maximization Problem

1. OBJECTIVE FUNCTION IS TO BE MAXIMIZED
2. ALL VARIABLES ARE NON-NEGATIVE $x \geq 0, y \geq 0, z \geq 0$
3. EACH CONSTRAINT HAS THE FORM $ax + by \leq c$

Definition 2: Slack Variables

For each inequality, introduce a slack variable to convert the inequality into an equation.

EX. $x + y \leq 16$

USE SLACK VARIABLE S_1

$$x + y + S_1 = 16$$

EX1

$$x + y \leq 16 \longrightarrow x + y + S_1 = 16$$
$$5x + 2y \leq 50 \longrightarrow 5x + 2y + S_2 = 50$$
$$y \leq 12 \qquad y + S_3 = 12$$

Definition 3: Simplex Tableau (Table)

1. Transform linear inequalities into linear equations with slack variables
2. Rewrite Objective Function
3. Create Simplex Table by creating a matrix like table containing the coefficients of each equation

Example 1

Set up the simplex table for

$$\text{Maximize: } P = 7x + 5y$$

$$\begin{aligned} x + y &\le 16 \\ 5x + 2y &\le 50 \\ y &\le 12 \\ x \ge 0, \quad & y \ge 0 \end{aligned}$$

(1) SLACK VARIABLES $x + y + S_1 = 16$

$5x + 2y + S_2 = 50$

$y + S_3 = 12$

(2) REWRITE $P = 7x + 5y$ AS $-7x - 5y + P = 0$

(3)

x	y	S_1	S_2	S_3	P	CONST
1	1	1	0	0	0	16
$\boxed{5}$	2	0	1	0	0	50
0	1	0	0	1	0	12
-7	-5	0	0	0	1	0

Definition 4: Simplex Method for Standard Maximization

1. Set up Initial Simplex Table
2. If all entries in last row are ≥ 0, then we are done.
3. If some of the entries in the bottom are negative
   (a) PIVOT COLUMN
      i. Find the column with the largest negative entry. If there is a tie, either will work.
      ii. This is the pivot column
   (b) PIVOT ROW
      i. Divide each entry in the constant column by the corresponding entry in the pivot column
      ii. Record the ratio to the right of that row
      iii. The row with the smallest POSITIVE ratio is the pivot row.
4. Pivot around the entry using the techniques from row reducing
   GO BACK TO STEP 2
5. Read the solution in the same way you would after row reducing a matrix.

Example 2

Brian has a small carpentry business that employs two carpenters and a finisher. They sell two types of tables: standard and amazing. Each standard table will result in a profit of \$50, and each amazing table results in a profit of \$54. A standard table requires 3 hours of carpentry and 1 hour of finishing. An amazing table requires 2 hours of carpentry and 2 hours of finishing. Each day there are 16 hours available for carpentry and 8 hours for finishing. How many tables of each type should be made to maximize profit?

MAXIMIZE $P = 50x + 54y$

SUBJECT TO
$$3x + 2y \le 16$$
$$x + 2y \le 8$$
$$x \ge 0,\ y \ge 0$$

(1) SLACK VARIABLES

$$3x + 2y + s_1 = 16$$
$$x + 2y + s_2 = 8$$

(2) REWRITE OBJECTIVE FUNCTION $P = 50x + 54y$ AS

$$-50x - 54y + P = 0$$

(3) TABLE

x	y	s_1	s_2	P	CONST	
3	2	1	0	0	16	$16/2 = 8$
1	2	0	1	0	8	$8/2 = 4$
-50	-54	0	0	1	0	

↑ PIVOT COLUMN

	x	y	s_1	s_2	P	~~CONST~~ CONST	
	3	2	1	0	0	16	$16/2=8$
ROW →	1	$\boxed{2}$	0	1	0	8	$8/2=4$
	-50	-54	0	0	1	0	

↑ PIVOT COLUMN

$\frac{1}{2}R_2 \rightarrow$

x	y	s_1	s_2	P	CONST
3	2	1	0	0	16
$1/2$	$\boxed{1}$	0	$1/2$	0	4
-50	54	0	0	1	0

$R_1 - 2R_2$
$R_3 + 54R_2$ $\rightarrow$

x	y	s_1	s_2	P	CONST	
$\boxed{2}$	0	1	-1	0	8	$8/2=4$
$1/2$	1	0	$1/2$	0	4	$4/(1/2)=8$
-23	0	0	27	1	216	

↑
PIVOT COLUMN

$\frac{1}{2}R_1 \rightarrow$

x	y	S_1	S_2	P	CONST
$\boxed{1}$	0	1/2	-1/2	0	4
1/2	1	0	1/2	0	4
-23	0	0	27	1	216

$R_2 - \frac{1}{2}R_1 \rightarrow$
$R_3 + 23R_1$

x	y	S_1	S_2	P	CONST
1	0	1/2	-1/2	0	4
0	1	-1/4	3/4	0	2
0	0	23/2	31/2	1	308

NO MORE NEGATIVES
SO WE ARE DONE

READ ANSWERS FROM TABLE USING THE BASIC COLUMNS (UNIT COLUMNS)

$x = 4, \; y = 2, \; P = \$308$

Example 3

Brian has a company that sells kitchen knives. The Basic Set consists of 2 utility and 1 chef's knife. The Regular Set consists of 2 utility, 1 chef's, and 1 slicer. The Deluxe Set consists of 3 utility, 1 chef's, and 1 slicer. Their profit is $30 for the Basic Set, $40 on a Regular Set, and $60 on a Deluxe Set. The factory has 800 utility, 400 chef's, and 200 slicers. How many of each type should be made in order to maximize profit?

X = # OF BASIC SETS, Y = # OF REGULAR, Z = # OF DELUXE

MAXIMIZE $P = 30x + 40y + 60z$

UTILITY: $2x + 2y + 3z \leq 800$

CHEF: $1x + 1y + 1z \leq 400$

SLICER: $1y + 1z \leq 200$

$x \geq 0, \; y \geq 0, \; z \geq 0$

SLACK VARIABLES:

$$2x + 2y + 3z + S_1 = 800$$
$$1x + 1y + 1z + S_2 = 400$$
$$1y + 1z + S_3 = 200$$
$$-30x - 40y - 60z + P = 0$$

TABLE

x	y	z	S_1	S_2	S_3	P	C
2	2	3	1	0	0	0	800
1	1	1	0	1	0	0	400
0	1	$\boxed{1}$	0	0	1	0	200
-30	-40	-60	0	0	0	1	0

$R_1 - 3R_3$
$R_2 - 1R_3$
$R_4 + 60R_3$

x	y	z	S_1	S_2	S_3	P	C
$\boxed{2}$	-1	0	1	0	-3	0	200
1	0	0	0	1	-1	0	200
0	1	$\boxed{1}$	0	0	1	0	200
-30	20	0	0	0	60	1	12000

$\frac{1}{2}R_1$

x	y	z	S_1	S_2	S_3	P	C
$\boxed{1}$	-1/2	0	1/2	0	-3/2	0	100
1	0	0	0	1	-1	0	200
0	1	1	0	0	1	0	200
-30	20	0	0	0	60	1	12000

$R_2 - R_1$
$R_4 + 30R_1$

x	y	z	S_1	S_2	S_3	P	C
1	-1/2	0	1/2	0	-3/2	0	100
0	1/2	0	-1/2	1	1/2	0	100
0	1	1	0	0	1	0	200
0	5	0	15	0	15	1	15000

BASIC (UNIT) COLUMNS ARE x, z, S_2, P
NON-BASIC (JUNK) ARE y, S_1, S_3

k

$x = 100, \quad y = 0, \quad z = 200$

100 BASIC SETS

0 REGULAR SETS

200 DELUXE SETS

MAX PROFIT OF \$15,000

$s_1 = 0$

$s_2 = 100$

$s_3 = 0$

UTILITY: $2x + 2y + 3z + s_1 = 800$

CHEF: $1x + y + z + s_2 = 400$

SLICER: $y + z + s_3 = 200$

0 UTILITY KNIFES LEFT

100 CHEF LEFT

0 SLICERS LEFT