MATH 210 FINITE MATHEMATICS

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3.3 Linear Programming: Graphical Solutions

Example 1

Consider the system of linear inequalities from our 3.2 - Example 1 word problem.

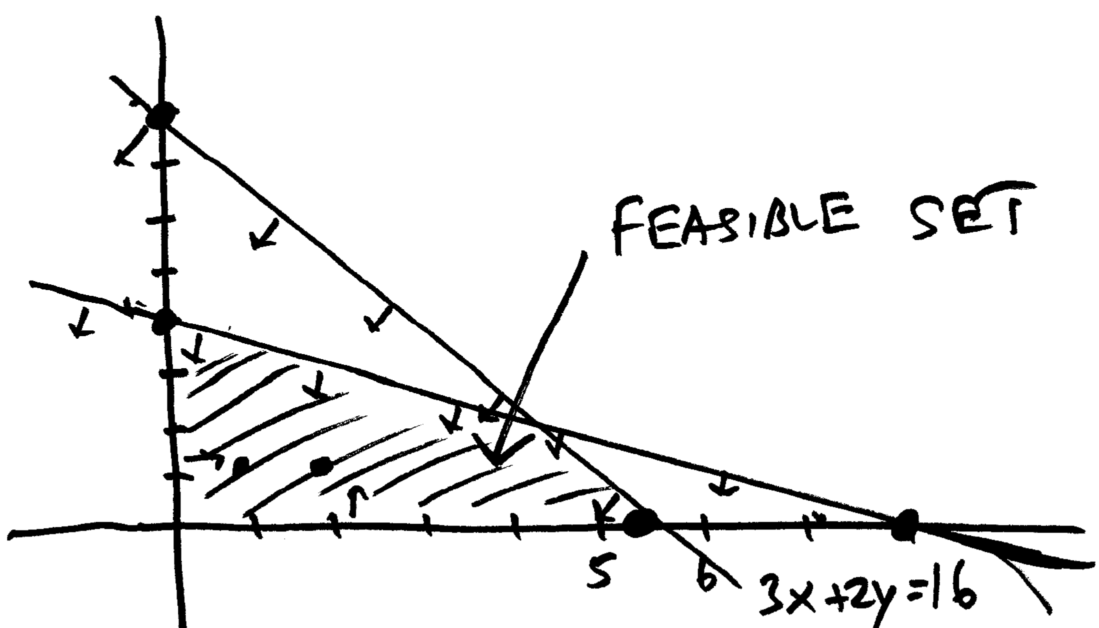
Maximize: $P = 50x + 54y$

Subject to:

$$\begin{aligned} 3x + 2y &\leq 16 \\ x + 2y &\leq 8 \\ x \geq 0, \quad & y \geq 0 \end{aligned}$$

LINE
$3x+2y=16$
$x+2y=8$

1. Sketch the feasible set (SHADED REGION)



2. Goal:

FIND A POINT IN THE FEASIBLE SET THAT MAXIMIZES $P=50x+54y$

Theorem 1: Corner Point Theorem

1. If a linear programming ~~program~~ PROBLEM has a solution, it must occur at a corner point of the feasible set.

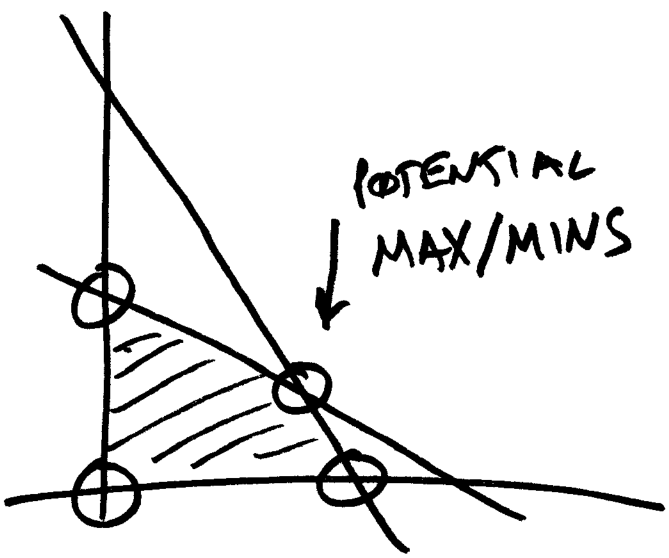
2. If the objective function P is optimized at two adjacent corners of the feasible set, then any point on the line connecting those points is considered an optimal solution.

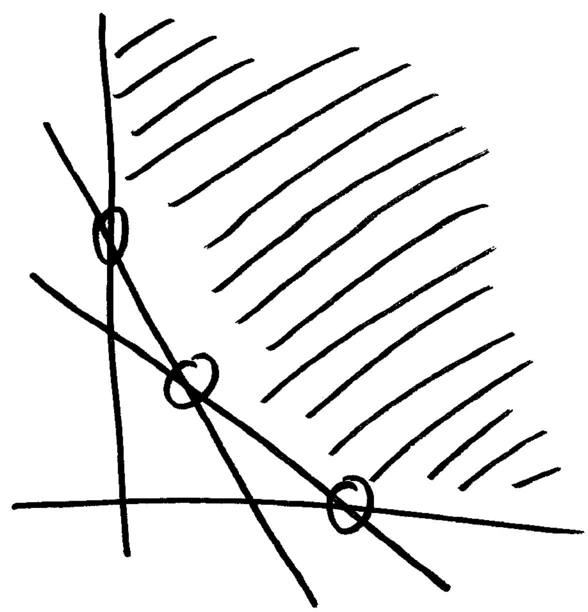
Steps 1

1. GRAPH THE FEASIBLE SET S
2. FIND COORDINATES OF ALL CORNERS OF S
3. EVALUATE THE OBJECTIVE FUNCTION AT EACH CORNER
4.

LARGEST VALUE IS THE MAX
SMALLEST IS THE MIN

5. IF THE MAX/MIN EXISTS IT MUST OCCUR AT A CORNER





Example 2

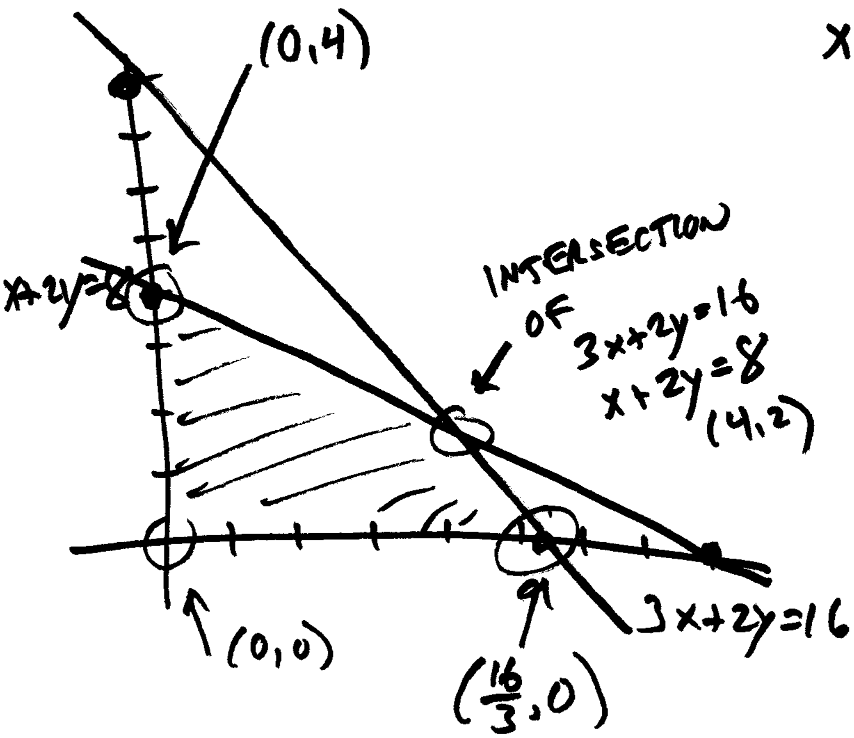
Brian has a small carpentry business that employs two carpenters and a finisher. They sell two types of tables: standard and amazing. Each standard table will result in a profit of \$50, and each amazing table results in a profit of \$54. A standard table requires 3 hours of carpentry and 1 hour of finishing. An amazing table requires 2 hours of carpentry and 2 hours of finishing. Each day there are 16 hours available for carpentry and 8 hours for finishing. How many tables of each type should be made to maximize profit?

MAXIMIZE $P = 50x + 54y$

SUBJECT TO $3x + 2y \le 16$

$x + 2y \le 8$

$x \ge 0, y \ge 0$



CORNERS	$50x+54y$
$(0,0)$	$P=0$
$(0,4)$	$P=216$
$(4,2)$	$P=308$
$(\frac{16}{3},0)$	$P=266.7$

MAX OF \$308 WHEN

$x=4$ (STANDARD TABLES)

$y=2$ (AMAZING TABLES)

Example 3

A finance company has \$120 million to invest in stocks S and T. Since stock T is riskier, management stipulated that the total amount invested in stock S be at least five times more than the amount invested in stock T. Stock T is expected to return an average of 20% and stock S an average of 12%. Determine the total amount that should be invested in each stock to maximize returns.

X = AMOUNT IN T, y = AMOUNT IN S

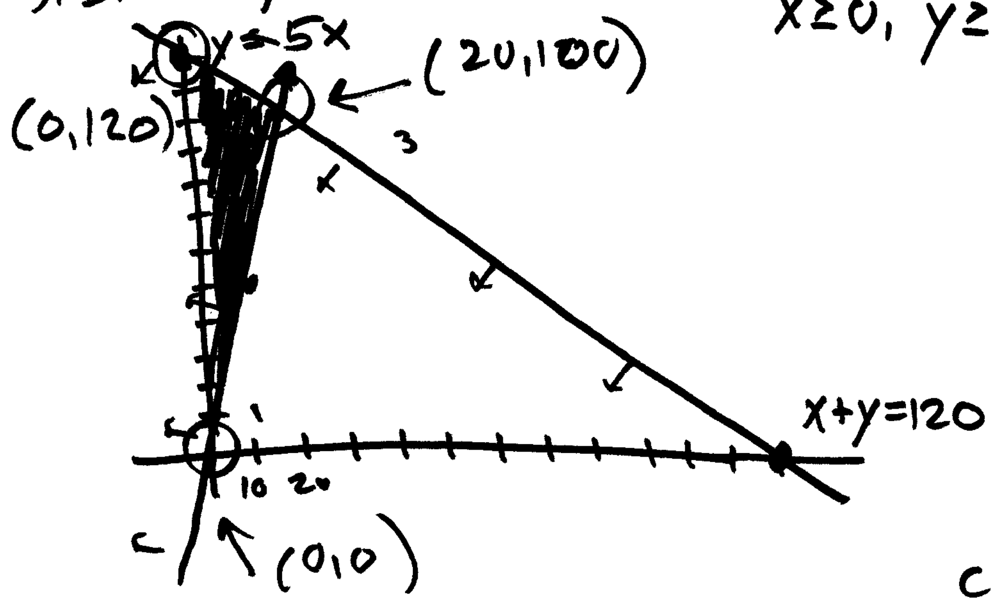
MAXIMIZE $R = .20x + .12y$

SUBJECT TO $x + y \le 120$

$y \ge 5x$

$x \ge 0,\ y \ge 0$

SKETCH $x+y=120$



SOLVE $\begin{cases} x+y=120 \\ y=5x \end{cases}$

SUBSTITUTION

$x + y = 120$

$x + 5x = 120$

$6x = 120$

$x = 20$

CORNERS	$R=.20x+.12y$
$(0,0)$	$R=0$
$(20,100)$	$R=16$
$(0,120)$	$R=14.4$

MAX RETURN OF \$16 MIL WHEN $x=\$20$, $y=\$100$

Example 4

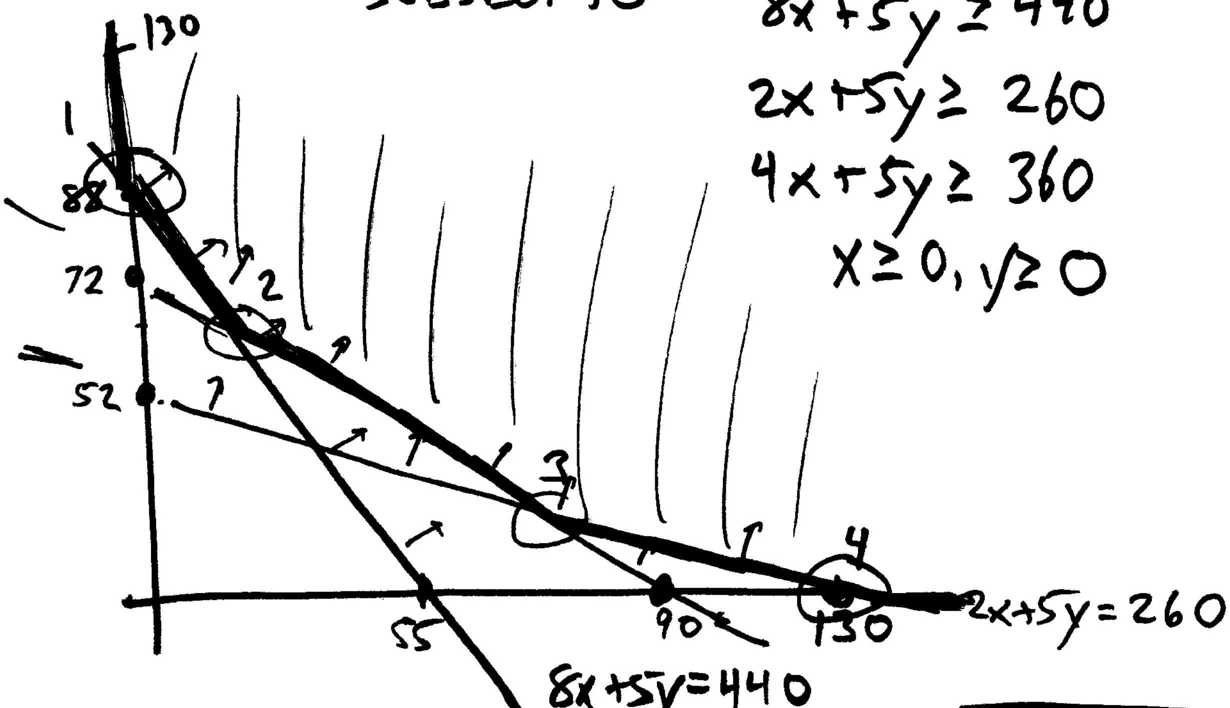
Brian uses two types of fertilizers. A 50-lb bag of Fertilizer A contains 8 lbs of nitrogen, 2 lbs of phosphorus, and 4 lbs of potassium. A 50-lb bag of Fertilizer B contains 5 lbs of nitrogen, 5 lbs of phosphorus, and 5 lbs of potassium. The minimum requirements for a field are 440 lbs of nitrogen, 260 lbs of phosphorus, and 360 lbs of potassium. If a 50-lb bag of A costs \$30 and a 50-lb bag of B costs \$20, find the amount of each type Brian should use to minimize cost while still meeting his requirements.

X = # OF FERTILIZER A, y = # OF B

MINIMIZE $C = 30x + 20y$

SUBJECT TO

$$8x + 5y \geq 440$$
$$2x + 5y \geq 260$$
$$4x + 5y \geq 360$$
$$x \geq 0,\ y \geq 0$$



(2) $8x + 5y = 440$
$4x + 5y = 360$
$x = 20$
$y = 56$

(3) $4x + 5y = 360$
$2x + 5y = 260$
$x = 50$
$y = 32$

CORNERS	30x+20y
(0,88)	\$1760
(20,56)	\$1720
(50,32)	\$2140
(130,0)	\$3900

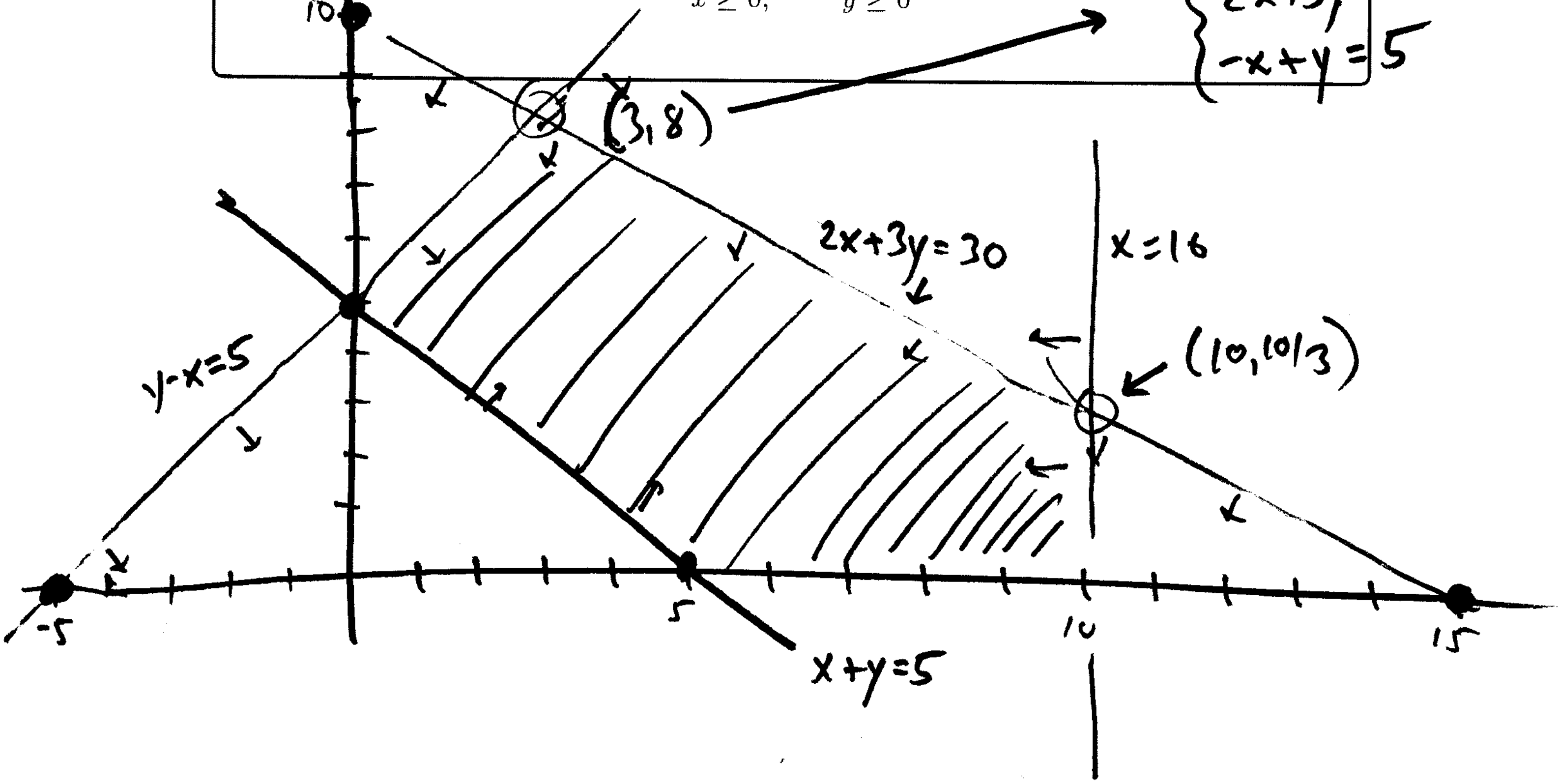
BUY 20 BAGS OF FERTILIZER A
AND 56 BAGS OF FERTILIZER B
FOR A COST OF $1720.

Example 5

Minimize: $P = 2x + 2y$

$$\begin{aligned} 2x + 3y &\le 30 \\ y - x &\le 5 \\ x + y &\ge 5 \\ x &\le 10 \\ x \ge 0, \quad & y \ge 0 \end{aligned}$$

SOLVE
$$\begin{cases} 2x+3y=30 \\ -x+y=5 \end{cases}$$



CORNERS	2x+2y
(0,5)	10
(5,0)	10
(10,0)	20
(10, 10/3)	26.7
(3,8)	22

MINIMUM OF 10
AT ANY POINT
ON THE LINE
$x+y=5$
$x \ge 0, y \ge 0$