MATH 210 FINITE MATHEMATICS

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3.2 Linear Programming Problems

Definition 1: LInear Programming Problem

1. Has a linear objective function to be maximized or minimized
2. Has constraints which the objective function is subjected to.
3. Constraints are linear inequalities or equalities

Example 1

Brian has a small carpentry business that employs two carpenters and a finisher. They sell two types of tables: standard and amazing. Each standard table will result in a profit of \$50, and each amazing table results in a profit of \$54. A standard table requires 3 hours of carpentry and 1 hour of finishing. An amazing table requires 2 hours of carpentry and 2 hours of finishing. Each day there are 16 hours available for carpentry and 8 hours for finishing. How many tables of each type should be made to maximize profit?

1. Variables? X=# OF STANDARD TABLES
   Y=# OF AMAZING TABLES
2. Constraints?

	STANDARD	AMAZING	RESTRICTION
CARPENTRY	3	2	16
FINISHING	1	2	8

$$3x + 2y \le 16$$
$$x + 2y \le 8$$

ADDITIONAL
$x \ge 0, y \ge 0$

3. Goal?

MAXIMIZE $P = 50x + 54y$

SUMMARY

MAXIMIZE $P = 50x + 54y$

SUBJECT TO $3x + 2y \le 16$

$x + 2y \le 8$

$x \ge 0,\ y \ge 0$

Example 2

A finance company has \$120 million to invest in stocks S and T. Since stock T is riskier, management stipulated that the total amount invested in stock S be at least five times more than the amount invested in stock T. Stock T is expected to return an average of 20% and stock S an average of 12%. Determine the total amount that should be invested in each stock to maximize returns.

(1) LET x = AMOUNT IN STOCK T

y = AMOUNT IN STOCK S

$S \geq 5 \cdot T$

(2) RESTRICTIONS

MONEY: $x + y \leq 120$

5 TIMES: $y \geq 5x$, $x \geq 0, y \geq 0$

(3) GOAL: MAXIMIZE $R = .20x + .12y$

SUMMARY

MAXIMIZE $R = .20x + .12y$

SUBJECT TO $x + y \leq 120$

$y \geq 5x$

$x \geq 0, y \geq 0$

Example 3

Brian uses two types of fertilizers. A 50-lb bag of Fertilizer A contains 8 lbs of nitrogen, 2 lbs of phosphorus, and 4 lbs of potassium. A 50-lb bag of Fertilizer B contains 5 lbs of nitrogen, 5 lbs of phosphorus, and 5 lbs of potassium. The minimum requirements for a field are 440 lbs of nitrogen, 260 lbs of phosphorus, and 360 lbs of potassium. If a 50-lb bag of A costs \$30 and a 50-lb bag of B costs \$20, find the amount of each type Brian should use to minimize cost while still meeting his requirements.

LET X = # OF BAG A , y = # OF BAG B

RESTRICTIONS

	A	B	MIN REQUIREMENT
NITROGEN	~~5~~ 8	5	440
PHOSPHORUS	2	5	260
POTASSIUM	4	5	360

$$\begin{cases} 8x + 5y \geq 440 \\ 2x + 5y \geq 260 \\ 4x + 5y \geq 360 \\ x \geq 0,\ y \geq 0 \end{cases}$$

GOAL: MINIMIZE $C = 30x + 20y$
SUBJECT TO

Example 4

Brian's company builds specialty gaming computers in two separate locations, Plant I and Plant II. The output at Plant I is at most 50 computers per month, whereas the output at Plant II is at most 70 per month. The computers are shipping to three stores, A, B, and C. Stores A, B, and C require a minimum of 30, 35, and 60 computers, respectively. Shipping costs from Plant I to A, B, and C are \$20, \$8, and \$10 per computer. Shipping costs from Plant II to each store is \$12, \$22, and \$18, respectively. What should the shipping schedule be if Brian wishes to meet the requirements of the stores and keep shipping costs to a minimum?