

MATH 210 FINITE MATHEMATICS

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2.6 Inverse of a Square Matrix

Definition 1: Inverse of Square Matrix

1. MATRIX MUST BE SQUARE $n \times n$
2. $A \cdot A^{-1} = I_n$ $A^{-1} \cdot A = I_n$
3. MATRIX WITH NO INVERSE IS
4. CALLED SINGULAR

Example 1

Show that the following two matrices are inverses.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$$

$$\begin{aligned} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix} &= \begin{bmatrix} 1(-2) + 2(3/2) & 1(1) + 2(-1/2) \\ 3(-2) + 4(3/2) & 3(1) + 4(-1/2) \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Ex] $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

FIND A^{-1}

START WITH $[A | I]$

$$= \left[\begin{array}{cc|cc} \textcircled{1} & 2 & 1 & 0 \\ -1 & 3 & 0 & 1 \end{array} \right]$$

NOW ROW REDUCE

$$\xrightarrow{R_2 + 1R_1} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 5 & 1 & 1 \end{array} \right]$$

$$\frac{1}{5}R_2 \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & \textcircled{1} & \frac{1}{5} & \frac{1}{5} \end{array} \right]$$

$$R_1 - 2R_2 \left[\begin{array}{cc|cc} 1 & 0 & \frac{3}{5} & -\frac{2}{5} \\ 0 & 1 & \frac{1}{5} & \frac{1}{5} \end{array} \right]$$

$$\text{FINAL: } A^{-1} = \begin{bmatrix} \frac{3}{5} & -\frac{2}{5} \\ \frac{1}{5} & \frac{1}{5} \end{bmatrix}$$

2x2 MATRIX SHORTCUT

GIVEN $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, THE INVERSE IS

$$A^{-1} = \frac{1}{D} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \quad \text{WHERE} \\ D = ad - bc$$

Ex] $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

$$\text{SO } A^{-1} = \frac{1}{1(3) - 2(-1)} \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3/5 & -2/5 \\ 1/5 & 1/5 \end{bmatrix}$$

NOTE: FOR A 2x2 MATRIX, IF ~~ad-bc=0~~
 $ad - bc = 0$, THEN THE MATRIX HAS
NO INVERSE.

~~0 + 12 = 12~~
~~12 + 12 = 24~~
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Steps 1: Finding the Inverse of a Matrix

1. Adjoin the $n \times n$ identity matrix I to obtain the augmented matrix

$$[A | I] \leftarrow \text{START HERE}$$

2. Use a sequence of row operations to reduce $[A|I]$ to

$$[I | B]$$

3. Matrix B is the inverse \longrightarrow so $B = A^{-1}$

4. If you don't end up with the identity matrix, then the inverse doesn't exist.

Example 2

Find the inverse to

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 3 & -3 \\ 2 & 1 & 1 \end{bmatrix}$$

START $\begin{bmatrix} \textcircled{1} & -1 & 1 & | & 1 & 0 & 0 \\ -1 & 3 & -3 & | & 0 & 1 & 0 \\ 2 & 1 & 1 & | & 0 & 0 & 1 \end{bmatrix}$

$R_2 + 1R_1$
 $R_3 - 2R_1$ $\begin{bmatrix} 1 & -1 & 1 & | & 1 & 0 & 0 \\ 0 & 2 & -2 & | & 1 & 1 & 0 \\ 0 & 3 & -1 & | & -2 & 0 & 1 \end{bmatrix}$

$\frac{1}{2}R_2$ $\begin{bmatrix} 1 & -1 & 1 & | & 1 & 0 & 0 \\ 0 & \textcircled{1} & -1 & | & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 3 & -1 & | & -2 & 0 & 1 \end{bmatrix}$

Continued $R_1 + 1R_2$
 $\xrightarrow{R_3 - 3R_2}$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/2 & 1/2 & 0 \\ 0 & 1 & -1 & 1/2 & 1/2 & 0 \\ 0 & 0 & 2 & -7/2 & -3/2 & 1 \end{array} \right]$$

$$\frac{1}{2}R_3 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/2 & 1/2 & 0 \\ 0 & 1 & -1 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1 & -7/4 & -3/4 & 1/2 \end{array} \right]$$

ROW 2 COLUMN 3 OF A^{-1}

$$\left(\begin{array}{l} R_2 + 1R_3 \\ \\ \end{array} \right) \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/2 & 1/2 & 0 \\ 0 & 1 & 0 & -5/4 & -1/4 & 1/2 \\ 0 & 0 & 1 & -7/4 & -3/4 & 1/2 \end{array} \right]$$

is $1/2$

Example 3

Does $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 3 & 3 & 5 \end{bmatrix}$ have an inverse?

AFTER REDUCING $\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 1 & 2 & 0 & 1 & 0 \\ 3 & 3 & 5 & 0 & 0 & 1 \end{array} \right]$

WE GET $\left[\begin{array}{ccc|ccc} 1 & 0 & 1/3 & -1/3 & 2/3 & 0 \\ 0 & 1 & 4/3 & 2/3 & -1/3 & 0 \\ 0 & 0 & 0 & -1 & -1 & -1 \end{array} \right]$

- row of 0s
- CAN'T HAVE AN INVERSE

Definition 2: Solving Systems of Equations with Inverses

Consider the following system:

$$2x + y + z = 1 \quad \leftarrow B$$

$$3x + 2y + z = 2$$

$$2x + y + 2z = -1$$

1. Convert to the form $AX = B$

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

2. Solution is

$$X = A^{-1}B$$

1. Set up matrices

$$X = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

2. Find A^{-1} SET UP $[A|I]$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 & 1 & 0 \\ 2 & 1 & 2 & 0 & 0 & 1 \end{array} \right]$$

THEN ROW REDUCE

$$\frac{1}{2}R_1 \left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 3 & 2 & 1 & 0 & 1 & 0 \\ 2 & 1 & 2 & 0 & 0 & 1 \end{array} \right]$$

$$\begin{array}{l} R_2 - 3R_1 \\ R_3 - 2R_1 \end{array} \left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & -\frac{3}{2} & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right]$$

$$2R_2 \left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & -1 & -3 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right]$$

$$R_1 - \frac{1}{2}R_2 \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 2 & -1 & 0 \\ 0 & 1 & -1 & -3 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right]$$

3. Solve for X

$$\begin{array}{l} R_1 - R_3 \\ R_2 + R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -1 & -1 \\ 0 & 1 & 0 & -4 & 2 & 1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right]$$

ANSWER: $X = A^{-1}B$

$$X = \begin{bmatrix} 3 & -1 & -1 \\ -4 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$X = \begin{bmatrix} 3(1) - 1(2) - 1(-1) \\ -4(1) + 2(2) + 1(-1) \\ -1(1) + 0(2) + 1(-1) \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$$

SOLUTION FOR SYSTEM IS

$$x = 2$$

$$y = -1$$

$$z = -2$$

2.2 TECHNIQUE IS ROW REDUCE

$$\begin{array}{c} \left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 3 & 2 & 1 & 2 \\ 2 & 1 & 2 & -1 \end{array} \right] \\ \curvearrowleft \text{MAGIC} \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{array} \right] \end{array}$$

But what if we have a new system?

$$2x + y + z = 2$$

$$3x + 2y + z = -3$$

$$2x + y + 2z = 1$$

BUT I CAN USE $X = A^{-1}B$

$$X = \begin{bmatrix} 3 & -1 & -1 \\ -4 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 \\ -13 \\ -1 \end{bmatrix}$$

$$x = 8$$

$$y = -13$$

$$z = -1$$