

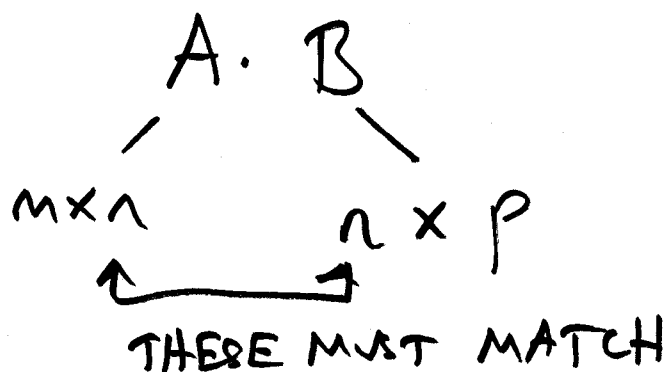
MATH 210 FINITE MATH

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2.5 Multiplication of Matrices

1. Order Matters $A \cdot B \neq B \cdot A$

2. You will need to compare the dimensions of the matrices



RESULTING AB MATRIX HAS SIZE $m \times p$

3. If matrix A has size 3×2 and B has size 2×5 , then AB HAS SIZE

$$\begin{array}{c} AB \\ \swarrow \quad \searrow \\ 3 \times 2 \quad 2 \times 5 \end{array}$$

SO AB IS 3×5

4. If matrix A has size 3×2 and B has size 2×5 , then

BA HAS SIZE?

$$\begin{array}{c} BA \\ \swarrow \quad \searrow \\ 2 \times 5 \quad 3 \times 2 \end{array}$$

DON'T MATCH

SO BA CAN'T EXIST

Example 1

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 5 \\ -1 & 4 \end{bmatrix}$, find

$$\underbrace{\begin{pmatrix} A \\ (2 \times 2) \end{pmatrix}} \underbrace{\begin{pmatrix} B \\ (2 \times 2) \end{pmatrix}}$$

AB
 2×2

1. $C = AB$

MULTIPLY AB

$$\begin{array}{l} \text{row 1} \\ \text{row 2} \end{array} \left[\begin{array}{cc} 1(2) + 2(-1) & ; & 1(5) + 2(4) \\ \text{---} & & \text{---} \\ 3(2) + 4(-1) & ; & 3(5) + 4(4) \end{array} \right]$$

$$= \begin{bmatrix} 0 & 13 \\ 2 & 31 \end{bmatrix}$$

2. $D = BA$ $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 5 \\ -1 & 4 \end{bmatrix}$

GOOD IDEA TO REWRITE AS

$$B = \begin{bmatrix} 2 & 5 \\ -1 & 4 \end{bmatrix}, A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2(1) + 5(3) & 2(2) + 5(4) \\ -1(1) + 4(3) & -1(2) + 4(4) \end{bmatrix}$$

$$= \begin{bmatrix} 17 & 24 \\ 11 & 14 \end{bmatrix}$$

Example 2

Let $A = \begin{bmatrix} 1 & -1 & 2 \\ -3 & -2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 3 \\ -2 & -1 \end{bmatrix}$
 find $C = AB$

$A \quad B$
 $2 \times 3 \quad 3 \times 2$
 $\underbrace{\hspace{10em}}$
 MATCHED

$$AB = \begin{bmatrix} 1 & -1 & 2 \\ -3 & -2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 3 \\ -2 & -1 \end{bmatrix} \quad 2 \times 2$$

$$= \left[\begin{array}{cc|cc} 1(1) + (-1)(0) + 2(-2) & 1(0) + (-1)(3) + 2(-1) & & \\ \hline -3(1) - 2(0) + 0(-2) & -3(0) - 2(3) + 0(-1) & & \end{array} \right]$$

$$= \begin{bmatrix} -3 & -5 \\ -3 & -6 \end{bmatrix}$$

WHICH OF THE FOLLOWING ARE TRUE?

T (1) AB HAS SIZE 2×2

T (2) BA HAS SIZE 3×3

F (3) # ROW 1 COLUMN 2 IS -6

T (4) BOTTOM LEFT ENTRY IS -3

A) 1 AND 2

B) 2 AND 4

C) 1, 2, 4

(d) 3

(e) NONE OF THE ABOVE

Can you multiple BA ?

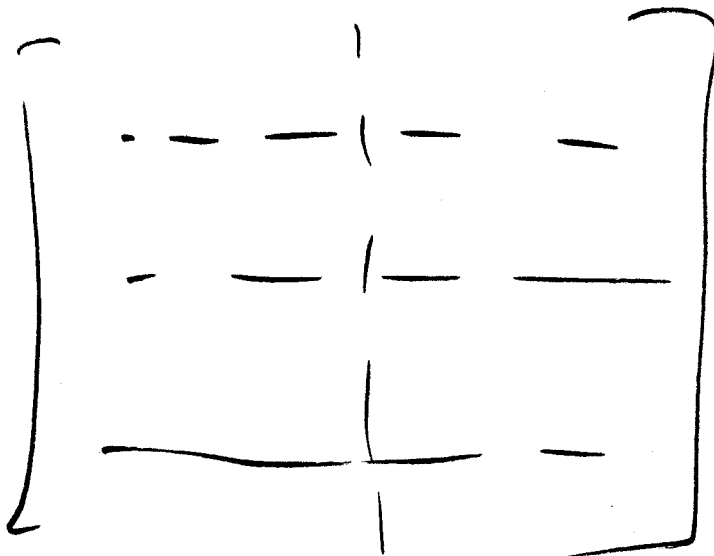
You try:

$$\text{a) } \begin{bmatrix} 1 & 0 & 6 \\ -1 & 2 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 5 & 4 \\ 8 & 6 \end{bmatrix} = \begin{bmatrix} 49 & 34 \\ -7 & -2 \end{bmatrix}$$

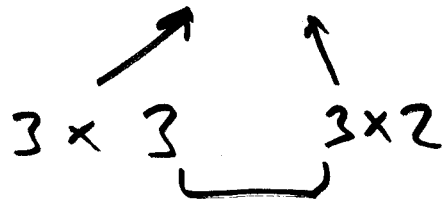
$\underbrace{\hspace{10em}}_{2 \times 3 \quad 3 \times 2}$

$$\text{b) } \begin{matrix} A & B \\ \begin{bmatrix} 3 & 1 & 4 \\ -1 & 0 & 2 \\ 0 & -2 & 3 \end{bmatrix} & \begin{bmatrix} 1 & -2 \\ 5 & 4 \\ 8 & 6 \end{bmatrix} \end{matrix} = \begin{bmatrix} 40 & 22 \\ 15 & 14 \\ 14 & 10 \end{bmatrix}$$

$\underbrace{\hspace{10em}}_{3 \times 3 \quad 3 \times 2}$



WHAT ABOUT $I \cdot A = A$



$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 4 \\ -2 & 5 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ -2 & 5 \\ 0 & 6 \end{bmatrix}$$

Definition 1: Laws of Matrix Multiplication

If the products and sums of matrices A , B , and C are defined, then

1. $(AB)C = A(BC)$ ASSOCIATIVE
2. $A(B+C) = AB+AC$ DISTRIBUTIVE

NOTE: MATRIX MULT IS NOT COMMUTATIVE $AB \neq BA$

Definition 2: Identity Matrix

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1. DIAGONAL OF 1s, REST ARE 0s
2. SIZE $n \times n$ (SQUARE MATRIX)
3. MULTIPLY ~~the~~ MATRIX A BY THE IDENTITY
 $A \cdot I_n = A$ AND $I_n \cdot A = A$

Example 3

Multiply $A = \begin{bmatrix} 1 & 4 \\ -2 & 5 \\ 0 & 6 \end{bmatrix}$ by the identity matrix. $A \cdot I = A$

3×2 2×2

$$\begin{bmatrix} 1 & 4 \\ -2 & 5 \\ 0 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ -2 & 5 \\ 0 & 6 \end{bmatrix} \quad \checkmark$$

Example 4

Perform the multiplication

$$\begin{matrix} 2 \times 2 & 2 \times 1 \\ \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \end{matrix} \rightarrow \boxed{AX = B}$$

$$\begin{bmatrix} 2x - 3y \\ x + 4y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad X = A^{-1} \cdot B$$

$$\Rightarrow \begin{matrix} 2x - 3y = 1 \\ x + 4y = 2 \end{matrix} \quad \leftarrow \begin{matrix} \text{SYSTEM} \\ \text{OF} \\ \text{EQUATIONS} \end{matrix}$$

Example 5

Write the following system of equations into matrix form

$$\begin{matrix} x + y + z = 6 \\ -2x + 5y + 7z = 1 \\ 4x - 3y + z = 2 \end{matrix}$$

$$\rightarrow \begin{matrix} \text{COEFFICIENTS} \\ \downarrow \\ [\quad] \cdot [\quad] = [\quad] \\ \downarrow \\ \text{VARIABLES} \\ \text{CONSTANTS} \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ -2 & 5 & 7 \\ 4 & -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 2 \end{bmatrix}$$