Math 210 Finite Math

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2.5 Multiplication of Matrices

1. Order Matters $A \cdot B \neq BA$

2. You will need to compare the dimensions of the matrices

   $A \cdot B$

   $m \times n$ $\quad$ $n \times p$

   THESE MUST MATCH

   RESULTING AB MATRIX HAS SIZE $m \times p$

3. If matrix A has size 3×2 and B has size 2×5, then AB HAS SIZE

   3×2 $\quad$ 2×5

   SO AB IS 3×5

4. If matrix A has size 3×2 and B has size 2×5, then BA HAS SIZE?

   2×5 $\quad$ 3×2

   DON'T MATCH

   SO BA CAN'T EXIST

b13 2.4 ADD #13-16

Example 1

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 5 \\ -1 & 4 \end{bmatrix}$, find

$$\underbrace{\overset{A}{(2 \times 2)} \quad \overset{B}{(2 \times 2)}}_{}$$

AB 2×2

1. $C = AB$ MULTIPLY AB

$$\begin{array}{c c} & \begin{array}{cc} \text{COLUMN 1} & \text{COLUMN 2} \end{array} \\ \begin{array}{c} \text{ROW 1} \\ \text{ROW 2} \end{array} & \begin{bmatrix} 1(2)+2(-1) & 1(5)+2(4) \\ 3(2)+4(-1) & 3(5)+4(4) \end{bmatrix} \end{array}$$

$$= \begin{bmatrix} 0 & 13 \\ 2 & 31 \end{bmatrix}$$

2. $D = BA$ $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 5 \\ -1 & 4 \end{bmatrix}$

GOOD IDEA TO REWRITE AS

$$B = \begin{bmatrix} 2 & 5 \\ -1 & 4 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2(1)+5(3) & 2(2)+5(4) \\ -1(1)+4(3) & -1(2)+4(4) \end{bmatrix}$$

$$= \begin{bmatrix} 17 & 24 \\ 11 & 14 \end{bmatrix}$$

Example 2

Let $A = \begin{bmatrix} 1 & -1 & 2 \\ -3 & -2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 3 \\ -2 & -1 \end{bmatrix}$

find $C = AB$

$$\underset{2\times 3}{A} \qquad \underset{3 \times 2}{B}$$

MATCHES

$$AB = \begin{bmatrix} 1 & -1 & 2 \\ -3 & -2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 3 \\ -2 & -1 \end{bmatrix} \qquad 2 \times 2$$

$$= \left[\begin{array}{c|c} 1(1) + -1(0) + 2(-2) & 1(0) - 1(3) + 2(-1) \\ \hline -3(1) - 2(0) + 0(-2) & -3(0) - 2(3) + 0(-1) \end{array}\right]$$

$$= \begin{bmatrix} -3 & -5 \\ -3 & -6 \end{bmatrix}$$

WHICH OF THE FOLLOWING ARE TRUE?

T (1) AB HAS SIZE 2×2

T (2) BA HAS SIZE 3×3

F (3) # ROW 1 COLUMN 2 IS -6

T (4) BOTTOM LEFT ENTRY IS -3

A) 1 AND 2

B) 2 AND 4

(C) 1,2,4

(d) 3

(e) NONE OF THE ABOVE

Can you multiple BA?

You try:

a) $\begin{bmatrix} 1 & 0 & 6 \\ -1 & 2 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 5 & 4 \\ 8 & 6 \end{bmatrix} = \begin{bmatrix} 49 & 34 \\ -7 & -2 \end{bmatrix}$

$2 \times 3 \quad 3 \times 2$

b) $\begin{array}{cc} A & B \\ \begin{bmatrix} 3 & 1 & 4 \\ -1 & 0 & 2 \\ 0 & -2 & 3 \end{bmatrix} & \cdot \begin{bmatrix} 1 & -2 \\ 5 & 4 \\ 8 & 6 \end{bmatrix} \end{array} = \begin{bmatrix} 40 & 22 \\ 15 & 14 \\ 14 & 10 \end{bmatrix}$

$3 \times 3 \quad 3 \times 2$

WHAT ABOUT $\underset{3 \times 3}{I} \cdot \underset{3 \times 2}{A} = A$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 4 \\ -2 & 5 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ -2 & 5 \\ 0 & 6 \end{bmatrix}$$

Definition 1: Laws of Matrix Multiplicaiton

If the products and sums of matrices A, B, and C are defined, then

1. $(AB)C = A(BC)$ ASSOCIATIVE
2. $A(B+C) = AB + AC$ DISTRIBUTIVE

NOTE: MATRIX MULT IS <u>NOT</u> COMMUTATIVE $AB \neq BA$

Definition 2: Identity Matrix

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1. DIAGONAL OF 1s, REST ARE 0s
2. SIZE $n \times n$ (SQUARE MATRIX)
3. MULTIPLY ~~an~~ MATRIX A BY THE IDENTITY

$$A \cdot I_n = A \quad \text{AND} \quad I_n \cdot A = A$$

Example 3

Multiply $A = \begin{bmatrix} 1 & 4 \\ -2 & 5 \\ 0 & 6 \end{bmatrix}$ by the identity matrix.

$$\underset{3\times 2}{A} \cdot \underset{2\times 2}{I} = A$$

$$\begin{bmatrix} 1 & 4 \\ -2 & 5 \\ 0 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ -2 & 5 \\ 0 & 6 \end{bmatrix} \checkmark$$

Example 4

Perform the multiplication

$$\underset{2\times 2}{\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}} \cdot \underset{2\times 1}{\begin{bmatrix} x \\ y \end{bmatrix}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \rightarrow \boxed{AX = B}$$

$$\begin{bmatrix} 2x - 3y \\ x + 4y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad X = A^{-1} \cdot B$$

$$\Rightarrow \begin{aligned} 2x - 3y &= 1 \\ 1x + 4y &= 2 \end{aligned} \leftarrow \text{SYSTEM OF EQUATIONS}$$

Example 5

Write the following system of equations into matrix form

$$\begin{aligned} x + y + z &= 6 \\ -2x + 5y + 7z &= 1 \\ 4x - 3y + z &= 2 \end{aligned} \rightarrow \underset{\text{COEFFICIENTS}}{[\ ]} \cdot \underset{\text{VARIABLES}}{[\ ]} = \underset{\text{CONSTANTS}}{[\ ]}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ -2 & 5 & 7 \\ 4 & -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 2 \end{bmatrix}$$