MATH 210 FINITE MATHEMATICS

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2.4 Matrices

Definition 1: Matrix

- ORDERED RECTANGULAR ARRAY OF NUMBERS
- M ROWS
- n COLUMNS
- SIZE $m \times n$
- ENTRY IN THE i^{TH} ROW j^{TH} COLUMN IS a_{ij}

Example 1

Consider the matrix

$$A = \begin{bmatrix} -3 & 5 & 1/3 & 6 \\ 9 & 21 & -2 & 0.2 \\ 1/2 & -9 & 16 & 0 \end{bmatrix}$$

1. How many rows? 3
2. How many columns? 4
3. Size? 3×4
4. Find
   (a) $a_{11} = -3$
   (b) $a_{34} = 0$
   (c) $a_{32} = -9$
   (d) $a_{21} = 9$
   (e) $a_{24} = 0.2$
   (f) $a_{13} = 1/3$

Definition 2: Row Matrix

SIZE [redacted] $1 \times n$

EX) 1×4 $\begin{bmatrix} 0 & -7 & 3 & 11/8 \end{bmatrix}$

Definition 3: Column Matrix

SIZE $m \times 1$

EX. 3×1 $\begin{bmatrix} -2 \\ 11/4 \\ 17 \end{bmatrix}$

Definition 4: Matrix Equality

TWO MATRICES ARE EQUAL IF THEY HAVE THE SAME SIZE AND THIER CORRESPONDING ENTRIES ARE THE SAME

Example 2

Find x, y, and z so that the following matrices are equal.

$$\begin{bmatrix} 1 & 2x-1 & 9 \\ 7 & z+1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 5 & 9 \\ y & 4 & 0 \end{bmatrix}$$

2×3

$1=1$ ✓ $2x-1=5$ $9=9$ ✓

$7=y$ $z+1=4$ $0=0$ ✓

SOLVE $2x-1=5$

$7=y$

$z+1=4$

$\rightarrow x=3, y=7, z=3$

Definition 5: Matrix Addition and Subtraction

1. MUST BE THE SAME SIZE
2. ADD OR SUBTRACT CORRESPONDING ENTRIES

Example 3

Perform the indicated operations

1. $\begin{bmatrix} 0 & 9 \\ 4 & 14 \\ -12 & 6 \end{bmatrix} + \begin{bmatrix} -8 & 0 \\ 5 & -10 \\ 7 & 100 \end{bmatrix} = \begin{bmatrix} 0+-8 & 9+0 \\ 4+5 & 14+-10 \\ -12+7 & 6+100 \end{bmatrix}$

$= \begin{bmatrix} -8 & 9 \\ 9 & 4 \\ -5 & 106 \end{bmatrix}$

2. $\begin{bmatrix} 12 & 2 & -9 \\ 1/2 & 0 & 12 \end{bmatrix} - \begin{bmatrix} -2 & 4 & 7 \\ 1/2 & -6 & 10 \end{bmatrix} = \begin{bmatrix} 12--2 & 2-4 & -9-7 \\ 1/2-1/2 & 0--6 & 12-10 \end{bmatrix}$

$= \begin{bmatrix} 14 & -2 & -16 \\ 0 & 6 & 2 \end{bmatrix}$

Definition 6: Transpose of a Matrix

- SWITCH ROWS AND COLUMNS
- TRANSPOSE OF A IS A^T
- ROW 1 BECOMES COLUMN 1
  ROW 2 BECOMES COLUMN 2
  AND SO ON

IF A HAS SIZE $m \times n$ THEN A^T HAS $n \times m$

Example 4

Find the transpose of

$$A = \begin{bmatrix} -6 & 0 \\ 15 & 0 \\ 1/2 & -3 \end{bmatrix}$$

3×2

↓ HAS SIZE 2×3

$$A^T = \begin{bmatrix} -6 & 15 & 1/2 \\ 0 & 0 & -3 \end{bmatrix}$$

Definition 7: Scalar Multiplication

1. MULTIPLY A MATRIX BY A NUMBER
2. SCALAR IS JUST A NUMBER
3. EACH ENTRY IS MULTIPLIED BY THE SCALAR

Example 5

If $M = \begin{bmatrix} 0 & 2 & 4 & -2 \\ 5 & -1 & 9 & 1/2 \\ 16 & -5 & 8 & 7 \end{bmatrix}$ find:

1. $2M = 2\begin{bmatrix} 0 & 2 & 4 & -2 \\ 5 & -1 & 9 & 1/2 \\ 16 & -5 & 8 & 7 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 4 & 8 & -4 \\ 10 & -2 & 18 & 1 \\ 32 & -10 & 16 & 14 \end{bmatrix}$

2. $-\frac{1}{3}M$

Example 6

Solve for u, x, y, and z.

$$\begin{bmatrix} x & -2 \\ 3 & y \end{bmatrix} - \begin{bmatrix} 2 & -z \\ 1 & -2 \end{bmatrix} = 2\begin{bmatrix} 2 & -1 \\ u & 2 \end{bmatrix}$$

$$\begin{bmatrix} x-2 & -2+z \\ 3-1 & y--2 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 2u & 4 \end{bmatrix}$$

$x-2=4$ $\quad -2+z=-2$ $\quad x=6$ $\quad z=0$

$2=2u$ $\quad y+2=4$ $\quad u=1$ $\quad y=2$