

# MATH 210 FINITE MATHEMATICS

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## 2.4 Matrices

### Definition 1: Matrix

- ORDERED RECTANGULAR ARRAY OF NUMBERS
- $m$  ROWS
- $n$  ~~ROWS~~ COLUMNS
- SIZE  $m \times n$
- ENTRY IN THE  $i$ TH ROW  $j$ TH COLUMN IS  $a_{ij}$

### Example 1

Consider the matrix

$$A = \begin{bmatrix} -3 & 5 & 1/3 & 6 \\ 9 & 21 & -2 & 0.2 \\ 1/2 & -9 & 16 & 0 \end{bmatrix}$$

1. How many rows? 3

2. How many columns? 4

3. Size?  $3 \times 4$

4. Find

(a)  $a_{11} = -3$

(c)  $a_{32} = -9$

(e)  $a_{24} = 0.2$

(b)  $a_{34} = 0$

(d)  $a_{21} = 9$

(f)  $a_{13} = 1/3$

**Definition 2: Row Matrix**SIZE ~~1 x n~~  $1 \times n$ EX)  $1 \times 4$   $[0 \ -7 \ 3 \ 1/8]$ **Definition 3: Column Matrix**SIZE  $m \times 1$ EX.  $3 \times 1$ 

$$\begin{bmatrix} -2 \\ 1/4 \\ 17 \end{bmatrix}$$

**Definition 4: Matrix Equality**

TWO MATRICES ARE EQUAL IF THEY HAVE THE SAME SIZE AND THEIR CORRESPONDING ENTRIES ARE THE SAME

**Example 2**Find  $x$ ,  $y$ , and  $z$  so that the following matrices are equal.

$$\begin{bmatrix} 1 & 2x-1 & 9 \\ 7 & z+1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 5 & 9 \\ y & 4 & 0 \end{bmatrix} \quad 2 \times 3$$

$$\begin{array}{l} 1=1 \checkmark \quad 2x-1=5 \quad 9=9 \checkmark \\ 7=y \quad z+1=4 \quad 0=0 \checkmark \end{array}$$

$$\begin{array}{l} \text{SOLVE } 2x-1=5 \\ 7=y \\ z+1=4 \end{array}$$

$$\rightarrow x=3, y=7, z=3$$

**Definition 5: Matrix Addition and Subtraction**

1. MUST BE THE SAME SIZE
2. ADD OR SUBTRACT CORRESPONDING ENTRIES

**Example 3**

Perform the indicated operations

$$1. \begin{bmatrix} 0 & 9 \\ 4 & 14 \\ -12 & 6 \end{bmatrix} + \begin{bmatrix} -8 & 0 \\ 5 & -10 \\ 7 & 100 \end{bmatrix} = \begin{bmatrix} 0 + -8 & 9 + 0 \\ 4 + 5 & 14 + -10 \\ -12 + 7 & 6 + 100 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & 9 \\ 9 & 4 \\ -5 & 106 \end{bmatrix}$$

$$2. \begin{bmatrix} 12 & 2 & -9 \\ 1/2 & 0 & 12 \end{bmatrix} - \begin{bmatrix} -2 & 4 & 7 \\ 1/2 & -6 & 10 \end{bmatrix} = \begin{bmatrix} 12 - -2 & 2 - 4 & -9 - 7 \\ 1/2 - 1/2 & 0 - -6 & 12 - 10 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & -2 & -16 \\ 0 & 6 & 2 \end{bmatrix}$$

**Definition 6: Transpose of a Matrix**

- SWITCH ROWS AND COLUMNS
  - ~~THE~~ TRANSPOSE OF  $A$  IS  $A^T$
  - ROW 1 BECOMES COLUMN 1  
ROW 2 BECOMES COLUMN 2  
AND SO ON
- IF  $A$  HAS SIZE  $m \times n$  THEN  $A^T$  HAS  $n \times m$

**Example 4**

Find the transpose of

$$A = \begin{bmatrix} -6 & 0 \\ 15 & 0 \\ 1/2 & -3 \end{bmatrix} \quad 3 \times 2$$

↓ HAS SIZE  $2 \times 3$

$$A^T = \begin{bmatrix} -6 & 15 & 1/2 \\ 0 & 0 & -3 \end{bmatrix}$$

**Definition 7: Scalar Multiplication**

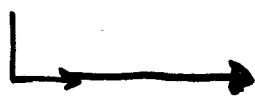
1. MULTIPLY A MATRIX BY A NUMBER
2. SCALAR IS JUST A NUMBER
3. EACH ENTRY IS MULTIPLIED BY THE SCALAR

**Example 5**

If  $M = \begin{bmatrix} 0 & 2 & 4 & -2 \\ 5 & -1 & 9 & 1/2 \\ 16 & -5 & 8 & 7 \end{bmatrix}$  find:

1.  $2M = 2 \begin{bmatrix} 0 & 2 & 4 & -2 \\ 5 & -1 & 9 & 1/2 \\ 16 & -5 & 8 & 7 \end{bmatrix}$

2.  $-\frac{1}{3}M$



$$\begin{bmatrix} 0 & 4 & 8 & -4 \\ 10 & -2 & 18 & 1 \\ 32 & -10 & 16 & 14 \end{bmatrix}$$

**Example 6**

Solve for  $u$ ,  $x$ ,  $y$ , and  $z$ .

$$\begin{bmatrix} x & -2 \\ 3 & y \end{bmatrix} - \begin{bmatrix} 2 & -z \\ 1 & -2 \end{bmatrix} = 2 \begin{bmatrix} 2 & -1 \\ u & 2 \end{bmatrix}$$

$$\begin{bmatrix} x-2 & -2+z \\ 3-1 & y-2 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 2u & 4 \end{bmatrix}$$

$$x-2=4$$

$$-2+z=-2$$

$$x=6 \quad z=0$$

$$2=2u$$

$$y+2=4$$

$$u=1 \quad y=2$$