MATH 210 FINITE MATHEMATICS

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2.3 System of Linear Equations - Under and Overdetermined Systems

Example 1

Consider the final reduced augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 17 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 13 \end{array}\right]$$

What does it mean? $x = 17$, $y = -3$, $0x + 0y + 0z = 13$ NO SOLUTION

SINCE THERE IS NO (x, y, z) THAT WORKS FOR $0x + 0y + 0z = 13$, THE SYSTEM HAS NO SOLUTION

LOOK FOR $\left[\begin{array}{cc|c} 0 & 0 & \text{NOT ZERO} \end{array}\right]$

Example 2

Consider the final reduced augmented matrix

$$\begin{array}{c} x\ y\ z \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 17 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

What does it mean?

$x = 17,\ y = -3,\ \underbrace{0x + 0y + 0z = 0}$

GIVES NO INFO ABOUT z

ANSWER:

$x = 17,\ y = -3,\ z =$ ANYTHING

FOR EX: $x = 17,\ y = -3,\ z = 4$

$x = 17,\ y = -3,\ z = 0$

UNDER DETERMINED SYSTEM

Example 3

Consider the final reduced augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 17 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

What does it mean?

Example 4

Solve the following system

$$\begin{aligned} x + y + 2z &= 3 \\ 3x - 2y + z &= 4 \\ 2x - 3y - z &= 1 \end{aligned}$$

PIVOT $\begin{bmatrix} 1 & 1 & 2 & | & 3 \\ 3 & -2 & 1 & | & 4 \\ 2 & -3 & -1 & | & 1 \end{bmatrix}$ $\xrightarrow[R_3 - 2R_1]{R_2 - 3R_1}$ $\begin{bmatrix} 1 & 1 & 2 & | & 3 \\ 0 & -5 & -5 & | & -5 \\ 0 & -5 & -5 & | & -5 \end{bmatrix}$

$R_3 - R_2$ $\begin{bmatrix} 1 & 1 & 2 & | & 3 \\ 0 & -5 & -5 & | & -5 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$

$-\frac{1}{5}R_2 \rightarrow$ $\begin{bmatrix} 1 & 1 & 2 & | & 3 \\ 0 & 1 & 1 & | & 1 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$

$R_1 - R_2 \rightarrow$ $\begin{bmatrix} 1 & 0 & 1 & | & 2 \\ 0 & 1 & 1 & | & 1 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$ ★ ROW REDUCED

Row 1: $x + z = 2$

Row 2: $y + z = 1$

Row 3: $z =$ ANYTHING

INFINITELY MANY SOLUTIONS

EX| $z = 1$ WHAT IS x, y?

$x + 1 = 2 \rightarrow x = 1$

$y + 1 = 1 \rightarrow y = 0$

$(2, \cancel{X}, 3)$ $(1, 0, 1)$

Example 5

Solve the following system

$$\begin{aligned} x + y &= 7 \\ 2x + 3y &= 8 \\ -5x - 5y &= -35 \end{aligned}$$

SET UP MATRIX

$$\left[\begin{array}{cc|c} 1 & 1 & 7 \\ 2 & 3 & 8 \\ -5 & -5 & -35 \end{array}\right] \quad \begin{array}{l} R_2 - 2R_1 \\ R_3 + 5R_1 \end{array} \quad \left[\begin{array}{cc|c} 1 & 1 & 7 \\ 0 & 1 & -6 \\ 0 & 0 & 0 \end{array}\right]$$

$2 - 2(1) = 0$
$3 - 2(1) = 1$
$8 - 2(7) = -6$

$$R_1 - R_2 \quad \left[\begin{array}{cc|c} 1 & 0 & 13 \\ 0 & 1 & -6 \\ 0 & 0 & 0 \end{array}\right]$$

1ST ROW: $x = 13$

2ND ROW $y = -6$

3RD ROW $0 = 0$

ANSWER: $x = 13$

$y = -6$

Example 6

Solve the following system

$$\begin{aligned} x + y &= 7 \\ 2x + 3y &= 8 \\ 4x - y &= 3 \end{aligned}$$

MATRIX

$$\left[\begin{array}{cc|c} 1 & 1 & 7 \\ 2 & 3 & 8 \\ 4 & -1 & 3 \end{array}\right] \quad \begin{array}{l} R_2 - 2R_1 \\ R_3 - 4R_1 \end{array} \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 7 \\ 0 & 1 & -6 \\ 0 & -5 & -25 \end{array}\right]$$

$$\begin{array}{l} R_1 - 1R_2 \\ R_3 + 5R_2 \end{array} \left[\begin{array}{cc|c} 1 & 0 & 13 \\ 0 & 1 & -6 \\ 0 & 0 & -55 \end{array}\right]$$

MEANS: $x = 13$

$y = -6$

$0 = -55$ ← BAD

NO SOLUTION

EX1 $4x - y - z = 4$

$8x + 2y - 2z = 8$

SOLVE THE SYSTEM

$$\left[\begin{array}{ccc|c} 4 & -1 & -1 & 4 \\ 8 & 2 & -2 & 8 \end{array}\right]$$

ROW REDUCE TO GET

$$\left[\begin{array}{ccc|c} 1 & 0 & -1/4 & 1 \\ 0 & 1 & 0 & 0 \\ x & y & z & c \end{array}\right]$$

$\left[\begin{array}{ccc|c} 1 & 0 & -1/4 & 1 \end{array}\right] \longrightarrow x - \frac{1}{4}z = 1$

$y = 0$

$z =$ ANY VALUE

INFINITELY MANY SOLUTIONS

IF $z = 8$ WHAT IS A SOLUTION?

$x - \frac{1}{4}z = x - \frac{1}{4}(8) = 1$

$x - 2 = 1$

$\boxed{x = 3}$

SOLUTION

$x = 3$
$y = 0$
$z = 8$

IS $x = 4, y = 0, z = 2$ A SOLUTION? NO