Math 210 Finite Mathematics

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2.2 System of Linear Equations - Unique Solutions

System of Equations **Augmented Matrix**

$$\begin{aligned} 3x - 2y &= 4 \\ 2x + 4y &= 8 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 2 & 4 & 6 & 22 \\ 2 & 8 & 5 & 27 \\ -1 & 1 & 2 & 2 \end{array}\right]$$

$$\begin{aligned} x + 2y + 3z &= 11 \\ 2y - 4z &= -6 \\ -x + y + 2z &= 2 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 17 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 13 \end{array}\right]$$

Definition 1: Row-Reduced Form of a Matrix

1. Each row consisting entirely of zeros must lie below rows having non-zero entries
2. The first non-zero entry in each row must be 1 (called a leading 1)
3. In any two successive (nonzero) rows, the leading 1 in the lower row lies to the right of the leading one in the upper row.
4. If a column contains a leading 1, then the other entries in that column must be zeros

Example 1

Determine which of the following matrices are in row-reduced form.

1. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 17 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 13 \end{array}\right]$

2. $\left[\begin{array}{ccc|c} 0 & 1 & 0 & 4 \\ 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 \end{array}\right]$

3. $\left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 2 & 7 \end{array}\right]$

4. $\left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{array}\right]$

Definition 2: Gauss-Jordan Method

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Definition 3: Unit Column

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Example 2

Solve the following system using Gauss Jordan Method

$$\begin{aligned} 3x + 5y &= 9 \\ 2x + 3y &= 5 \end{aligned}$$

Example 3

Solve

$$\begin{aligned} 2y + 3z &= 7 \\ 3x + 6y - 12z &= -3 \\ 5x - 2y + 2z &= -7 \end{aligned}$$

Example 4

Brian wants to buy a 50 items from Amazon (board games and DVDs). The board games cost \$35 and the DVDs cost \$20. With a maximum of \$1600, how many board games and DVDs can he buy? Set up only.