Quiz 9 Math 232, Calculus III, Fall 2016 Name:

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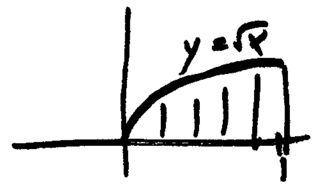
1. Evaluate $\iiint_E 6xy\, dV$, where E lies under the plane $z = 1 + x + y$ and above the region in the xy-plane bounded by the curves $y = \sqrt{x}$, $y = 0$, and $x = 1$.

$$\iint_D \int_{z=0}^{z=1+x+y} 6xy\, dz\, dA$$

D: 

$$\begin{cases} 0 \le x \le 1 \\ 0 \le y \le \sqrt{x} \end{cases}$$

$$\int_0^1 \int_0^{\sqrt{x}} \int_0^{1+x+y} 6xy\, dz\, dy\, dx$$

INSIDE: $\int_0^{1+x+y} 6xy\, dz = 6xyz\Big|_0^{1+x+y} = 6xy(1+x+y) - 0$

$= 6xy + 6x^2y + 6xy^2$

MIDDLE: $\int_0^{\sqrt{x}} 6xy + 6x^2y + 6xy^2\, dy = 3xy^2 + 3x^2y^2 + 2xy^3\Big|_0^{\sqrt{x}}$

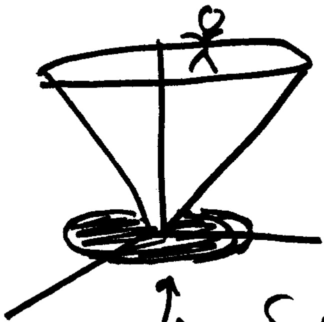
$= 3x^2 + 3x^3 + 2x^{5/2}$

OUTSIDE: $\int_0^1 3x^2 + 3x^3 + 2x^{5/2}\, dx$

$= x^3 + \frac{3}{4}x^4 + \frac{4}{7}x^{7/2}\Big|_0^1$

$= \left(1 + \frac{3}{4} + \frac{4}{7}\right) - (0 + 0 + 0) = \frac{65}{28}$

2. Evaluate $\int\int\int_E x + y \, dV$ where E is the region lies inside $x^2 + y^2 = 16$ bounded by the xy-plane and $z = \sqrt{x^2 + y^2}$.



$D = \begin{cases} 0 \le \theta \le 2\pi \\ 0 \le r \le 4 \end{cases}$ AND $0 \le z \le \sqrt{x^2+y^2}$

CHANGE TO POLAR TO GET

$0 \le z \le \sqrt{r^2} = r$

$$\int_0^{2\pi}\int_0^4\int_0^r (r\cos\theta + r\sin\theta)\, r \, dz\, dr\, d\theta$$

INSIDE: $\int_0^r r^2(\cos\theta + \sin\theta)\, dz = r^2(\cos\theta + \sin\theta)\, z\Big|_0^r$

$= r^3(\cos\theta + \sin\theta)$

MIDDLE: $\int_0^4 r^3(\cos\theta+\sin\theta)\, dr = \frac{1}{4} r^4 (\cos\theta + \sin\theta)\Big|_0^4$

$= 64(\cos\theta + \sin\theta)$

OUTSIDE: $\int_0^{2\pi} 64(\cos\theta + \sin\theta)\, d\theta = 64(\sin\theta - \cos\theta)\Big|_0^{2\pi}$

$= 64(\sin 2\pi - \cos 2\pi) - 64(\sin 0 - \cos 0)$

$= 64(0-1) - 64(0-1)$

$= \boxed{0}$