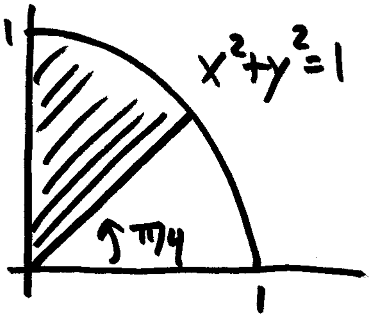
Quiz 8 Math 232, Calculus III, Spring 2016 Name: SOLUTION

Show all your work to receive full credit.

1. Show $\iint_D 2x - y\,dA = \frac{2}{3} - \frac{\sqrt{2}}{2}$ where D is the region in the first quadrant enclosed by the circle $x^2 + y^2 = 1$ and the lines $y = x$ and $x = 0$.



$$D = \{(r,\theta) \mid \pi/4 \le \theta \le \pi/2,\ 0 \le r \le 1\}$$

$$\iint_D 2x-y\,dA = \int_{\pi/4}^{\pi/2}\int_0^1 (2r\cos\theta - r\sin\theta)\,r\,dr\,d\theta$$

INSIDE: $\int_0^1 (2r\cos\theta - r\sin\theta)\,r\,dr = \int_0^1 r^2(2\cos\theta - \sin\theta)\,dr$

$$= \frac{1}{3}r^3(2\cos\theta - \sin\theta)\Big|_{r=0}^{r=1}$$

$$= \frac{2}{3}\cos\theta - \frac{1}{3}\sin\theta$$

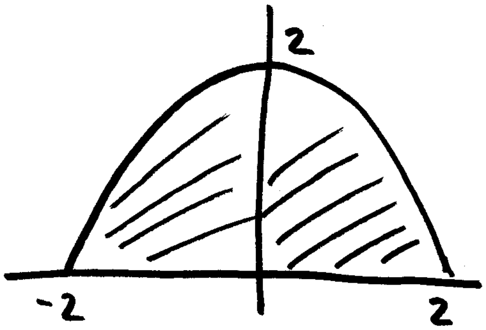
OUTSIDE: $\int_{\pi/4}^{\pi/2} \frac{2}{3}\cos\theta - \frac{1}{3}\sin\theta\,d\theta = \frac{2}{3}\sin\theta + \frac{1}{3}\cos\theta\Big|_{\theta=\pi/4}^{\theta=\pi/2}$

$$= \left[\frac{2}{3}\sin\pi/2 + \frac{1}{3}\cos\pi/2\right] - \left[\frac{2}{3}\sin\pi/4 + \frac{1}{3}\cos\pi/4\right]$$

$$= \left[\frac{2}{3} + 0\right] - \left[\frac{\sqrt{2}}{3} + \frac{\sqrt{2}}{6}\right]$$

$$= \frac{2}{3} - \frac{\sqrt{2}}{2}$$

2. Evaluate $\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}} \cos(x^2+y^2)\ dy\ dx$ by first converting the double integral to polar coordinates.



$$D = \{ (r,\theta) \mid 0 \le \theta \le \pi,\ 0 \le r \le 2 \}$$

$$\iint_D \cos(x^2+y^2)\,dA = \int_0^{\pi}\int_0^{2} \cos(r^2)\, r\, dr\, d\theta$$

INSIDE : $\int_0^2 r\cos(r^2)\,dr$

LET $u = r^2$, $du = 2r\,dr \Rightarrow \frac{1}{2}du = r\,dr$

IF $r=2$, $u = 2^2 = 4$

IF $r=0$, $u = 0^2 = 0$

$$\int_0^4 \frac{1}{2}\cos(u)\,du = \frac{1}{2}\sin(u)\Big|_{u=0}^{u=4} = \frac{1}{2}\sin 4 - \frac{1}{2}\sin 0 = \frac{1}{2}\sin 4$$

OUTSIDE :

$$\int_0^{\pi} \frac{1}{2}\sin 4\, d\theta = \frac{1}{2}(\sin 4)\theta\Big|_{\theta=0}^{\theta=\pi} = \frac{1}{2}(\sin 4)\pi - \frac{1}{2}(\sin 4)\cdot 0$$

$$= \frac{\pi}{2}\sin 4$$