QUIZ 7 MATH 232, CALCULUS III, SPRING 2016 NAME: SOLUTION

Show all your work to receive full credit.

1. Evaluate $\int_{-3}^{3}\int_{0}^{\pi/2} y + y^2\cos(x)\,dx\,dy$.

(1) $\int_0^{\pi/2} y + y^2\cos(x)\,dx = xy + y^2\sin x\Big|_0^{\pi/2} = \left[\frac{\pi}{2}\cdot y + y^2\sin\pi/2\right] - [0+0]$

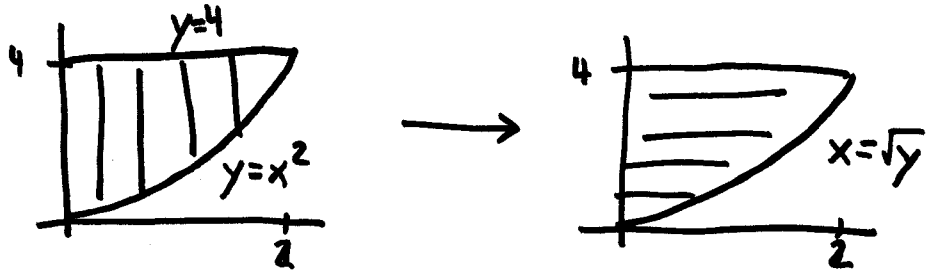
$= \frac{\pi}{2}y + y^2$

(2) $\int_{-3}^{3} \frac{\pi}{2}y + y^2\,dy = \frac{\pi}{4}y^2 + \frac{1}{3}y^3\Big|_{-3}^{3}$

$= \left[\frac{\pi}{4}\cdot(3)^2 + \frac{1}{3}(3)^3\right] - \left[\frac{\pi}{4}(-3)^2 + \frac{1}{3}(-3)^3\right] = \frac{9\pi}{4} + 9 - \frac{9\pi}{4} + 9$

$= 18$

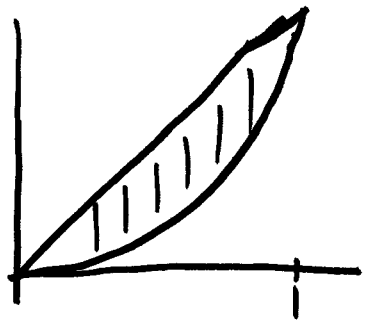
2. Sketch the region of integration and change the order of integration of $\int_0^2\int_{x^2}^4 \sqrt{y}\sin(y)\,dy\,dx$. Do not evaluate.



$\int_0^4\int_0^{\sqrt{y}} \sqrt{y}\sin y\,dx\,dy$

3. Sketch the region D and evaluate $\int\int_D x^2 + 2y \, dA$ where D is bounded by $y = x$, $y = x^3$, and $x \geq 0$.

D



$D = \{(x,y) \mid 0 \leq x \leq 1, \; x^3 \leq y \leq x\}$

$$\int_0^1 \int_{x^3}^{x} x^2 + 2y \, dy \, dx$$

(1) $$\int_{x^3}^{x} x^2 + 2y \, dy = xy + y^2 \Big|_{x^3}^{x} = \left[x^2 \cdot x + x^2\right] - \left[x^2 \cdot x^3 + (x^3)^2\right]$$

$$= x^3 + x^2 - x^5 - x^6$$

(2) $$\int_0^1 x^3 + x^2 - x^5 - x^6 \, dx = \frac{1}{4}x^4 + \frac{1}{3}x^3 - \frac{1}{6}x^6 - \frac{1}{7}x^7 \Big|_0^1$$

$$= \left[\frac{1}{4} + \frac{1}{3} - \frac{1}{6} - \frac{1}{7}\right] - [0]$$

$$= \frac{23}{84}$$