QUIZ 4 MATH 232, CALCULUS III, SPRING 2016 NAME: SOLUTION

Show all your work to receive full credit.

1. Find the parametric equations for the tangent line to the curve with the given parametric equations

$$x = \ln(t+1),\ y = [redacted]\cos(2t),\ z = 2^t$$

at the point (0, [redacted], 1).

• DIRECTION VECTOR OF TANGENT LINE IS $\left\langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right\rangle$

$$\left\langle \frac{1}{t+1}, -2\sin(2t), 2^t \ln 2 \right\rangle$$

• FIND t-VALUE AT POINT (0, [redacted]): $t=0$

• $\vec{v} = \left\langle \frac{1}{0+1}, -2\sin(0), 2^0 \ln 2 \right\rangle = \langle 1, 0, \ln 2 \rangle$

• TANGENT LINE: $r(t) = \langle 1, 0, \ln 2 \rangle t + \langle 0, 1, 1 \rangle$

$$x = 1t,\quad y = 0t + 1,\quad z = (\ln 2)t + 1$$

2. Set up the integral for the length of the curve defined by $r(\vec{t}) = (\cos t)i + (\sin t)j + (\ln\cos t)k$ for $0 \le t \le \pi/4$. **Extra Credit:** Find the value of the arc length.

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt \quad , \quad \frac{dx}{dt} = \sin t$$

$$\frac{dy}{dt} = -\cos t$$

$$= \int_0^{\pi/4} \sqrt{(\sin t)^2 + (-\cos t)^2 + (-\tan t)^2}\, dt \qquad \frac{dz}{dt} = \frac{1}{\cos t} \cdot -\sin t = -\tan t$$

$$= \int_0^{\pi/4} \sqrt{1 + \tan^2 t}\, dt$$

EXTRA CREDIT

$$= \int_0^{\pi/4} \sqrt{\sec^2 t}\, dt = \int_0^{\pi/4} \sec t\, dt = \ln|\sec t + \tan t| \Big|_0^{\pi/4}$$

$$= \ln|\sec \pi/4 + \tan \pi/4| - \ln|\sec 0 + \tan 0|$$

$$= \ln|\sqrt{2} + 1| - \ln|1|$$

$$= \ln|\sqrt{2} + 1|$$

3. Let $\vec{r(t)} = 33t^2\mathbf{i} + \sin(\pi t)\mathbf{j} + \dfrac{t}{\sqrt{t^2-1}}\mathbf{k}$. Find $\displaystyle\int_1^2 \vec{r}(t)\,dt$.

$$\int_1^2 33t^2 i + \sin(\pi t) j + \frac{t}{\sqrt{t^2-1}} k \; dt$$

- $$\int_1^2 33t^2 i \, dt = 11t^3 i \Big|_1^2 = 11(2)^3 i - 11(1)^3 i = 88i - 11i = 77i$$

- $$\int_1^2 \sin(\pi t) j \, dt = -\frac{\cos(\pi t)}{\pi} j \Big|_1^2 = -\frac{\cos(2\pi)}{\pi} j + \frac{\cos(\pi)}{\pi} j = \frac{-1}{\pi} j - \frac{1}{\pi} j = \frac{-2}{\pi} j$$

- $$\int_1^2 \frac{t}{\sqrt{t^2-1}} k \, dt.$$ USE U-SUB

LET $u = t^2 - 1$, $du = 2t\,dt \rightarrow \frac{1}{2} du = t\,dt$

IF $t = 2$, $u = 3$

IF $t = 1$, $u = 0$

$$\Rightarrow \int_0^3 \frac{1}{2} \cdot \frac{1}{\sqrt{u}} du = \int_0^3 \frac{1}{2\sqrt{u}} du = \sqrt{u} \Big|_0^3 = \sqrt{3} - \sqrt{0} = \sqrt{3}$$

FINAL: $$\int_1^2 r(t)dt = 77i - \frac{2}{\pi} j + \sqrt{3} k$$

OR $\langle 77, -\frac{2}{\pi}, \sqrt{3} \rangle$