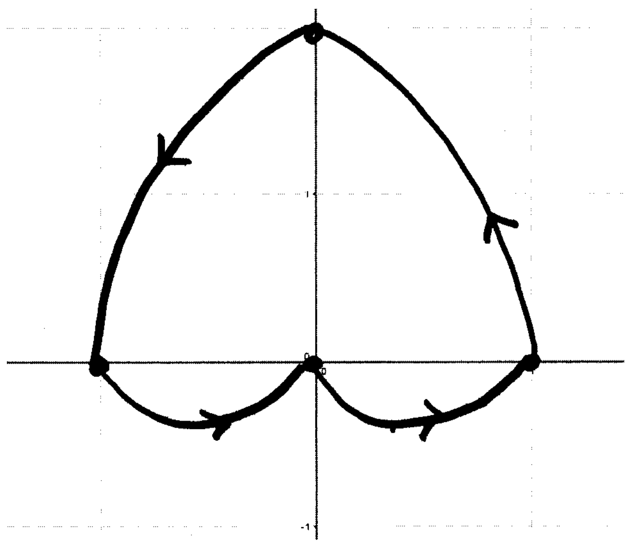
QUIZ 2 MATH 232, CALCULUS III, FALL 2016 NAME: SOLUTION

Show all work for full credit.

1. Sketch the polar curve: $r = 1 + \sin(\theta)$



θ	r
0	1
$\frac{\pi}{2}$	2
π	1
$3\pi/2$	0
$\frac{5\pi}{4}$	$1+-\frac{\sqrt{2}}{2} \approx .3$
$\frac{2\pi}{4}$	$1+-\frac{\sqrt{2}}{2} \approx .3$

2. Set up the integral for the length of the curve $r = 1 + \sin(\theta)$

$$\frac{dr}{d\theta} = \cos\theta$$

$$L = \int_0^{2\pi} \sqrt{(1+\sin\theta)^2 + (\cos\theta)^2}\, d\theta$$

Identites:

$$\cos^2(\theta) = \frac{1}{2}(1 + \cos(2\theta))$$

$$\sin^2(\theta) = \frac{1}{2}(1 - \cos(2\theta))$$

3. Set up the integral of the region bounded by $r = 1 + \sin(\theta)$

$$A = \int_0^{2\pi} \frac{1}{2}(1+\sin\theta)^2 d\theta$$

$$= \int_0^{2\pi} \frac{1}{2}(1+2\sin\theta+\sin^2\theta)\,d\theta$$

$$= \int_0^{2\pi} \frac{1}{2} + \sin\theta + \frac{1}{2}\sin^2\theta\,d\theta$$

4. Find the value of the area bounded by the region $r = 1 + \sin(\theta)$.

$$(1)\ \int_0^{2\pi} \frac{1}{2}\,d\theta = \frac{1}{2}\theta\Big|_0^{2\pi} = \frac{1}{2}(2\pi) - \frac{1}{2}(0) = \pi$$

$$(2)\ \int_0^{2\pi} \sin\theta\,d\theta = -\cos\theta\Big|_0^{2\pi} = -\cos 2\pi + \cos 0 = 0$$

$$(3)\ \int_0^{2\pi} \frac{1}{2}\sin^2\theta\,d\theta = \int_0^{2\pi} \frac{1}{4}(1-\cos 2\theta)\,d\theta$$

$$= \int_0^{2\pi} \frac{1}{4} - \frac{1}{4}\cos 2\theta\,d\theta$$

$$= \frac{1}{4}\theta - \frac{1}{8}\sin 2\theta\Big|_0^{2\pi}$$

$$= \left[\frac{1}{4}(2\pi) - \frac{1}{8}\sin 4\pi\right] - \left[\frac{1}{4}(0) - \frac{1}{8}\sin 0\right]$$

$$= \frac{\pi}{2}$$

TOTAL: $\pi + 0 + \frac{\pi}{2} = \frac{3\pi}{2}$