QUIZ 1 MATH 232, CALCULUS III, SPRING 2018 NAME: SOLUTION

1. Sketch the curve defined by the parametric equations

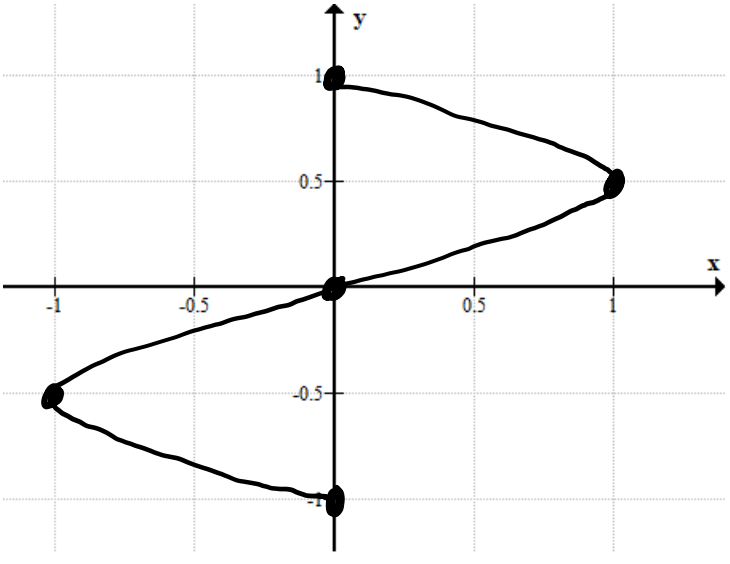
$$x = \sin(t),\ y = \frac{t}{\pi},\ -\pi \le t \le \pi$$

by using 5 t-values. Start at $t = -\pi$ and increase by $\pi/2$. Note the direction of the curve using small arrows.

(a) Make a T-table.

t	$x = \sin(t)$	$y = \frac{t}{\pi}$	POINTS
$-\pi$	$\sin(-\pi) = 0$	$\frac{-\pi}{\pi} = -1$	$(0, -1)$
$-\pi/2$	$\sin(-\pi/2) = -1$	$\frac{-\pi/2}{\pi} = -\frac{1}{2}$	$(-1, -\frac{1}{2})$
0	$\sin 0 = 0$	$\frac{0}{\pi} = 0$	$(0, 0)$
$\pi/2$	$\sin(\pi/2) = 1$	$\frac{\pi/2}{\pi} = \frac{1}{2}$	$(1, \frac{1}{2})$
π	$\sin(\pi) = 0$	$\frac{\pi}{\pi} = 1$	$(0, 1)$

(b) Sketch



(c) Find $\frac{dy}{dx}$. Then find the slope at $t = \pi/3$.

- $\frac{dx}{dt} = \cos(t)$, $\frac{dy}{dt} = \frac{1}{\pi}$
- $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{1}{\pi}}{\cos(t)} = \frac{1}{\pi\cos(t)}$
- $\left.\frac{dy}{dx}\right|_{t=\pi/3} = \frac{1}{\pi\cos(\pi/3)} = \frac{1}{\pi \cdot \frac{1}{2}} = \frac{2}{\pi}$

(d) Find all horizontal and vertical tangents.

HORIZONTAL WHEN $\frac{dy}{dt} = 0$: $\frac{1}{\pi} \neq 0$

NO HORIZONTAL TANGENTS

VERTICAL WHEN $\frac{dx}{dt} = 0$: $\cos(t) = 0$

$t = -\frac{\pi}{2}, \frac{\pi}{2}, \ldots$

ONLY $-\frac{\pi}{2}$ AND $\frac{\pi}{2}$ ARE IN THE DOMAIN

$t = -\pi/2$: $(-1, -\frac{1}{2})$

$t = \pi/2$: $(1, \frac{1}{2})$