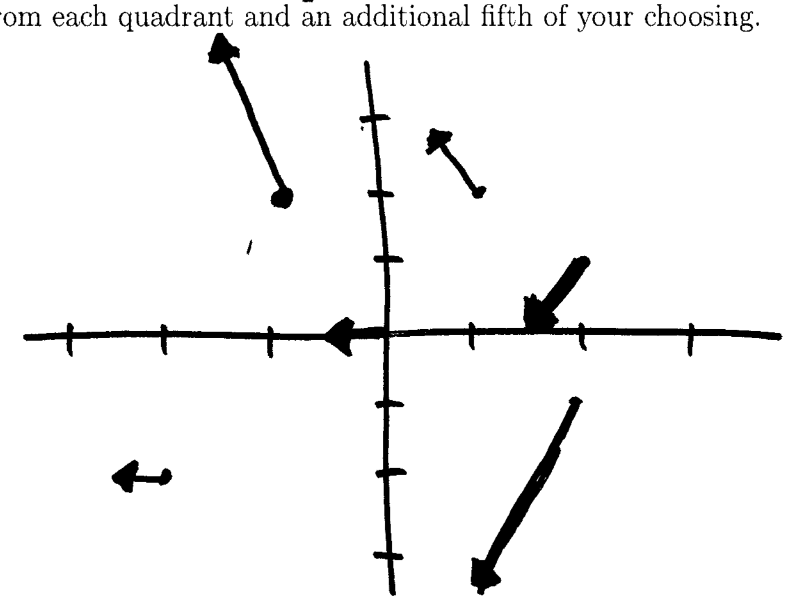
QUIZ 11 MATH 232, CALCULUS III, FALL 2016 NAME: SOLUTION

Show all your work to receive full credit.

1. Sketch the vector field $F(x, y) = -\frac{1}{2}\mathbf{i} + (y - x)\mathbf{j}$ by plotting 5 points/vectors. Choose one point from each quadrant and an additional fifth of your choosing.

POINT	
$(0,0)$	$\langle -1/2, 0\rangle$
$(2,1)$	$\langle -1/2, -1\rangle$
$(1,2)$	$\langle -1/2, 1\rangle$
$(-1,2)$	$\langle -1/2, 3\rangle$
$(-2,-2)$	$\langle -1/2, 0\rangle$
$(2,-1)$	$\langle -1/2, -3\rangle$



2. Evaluate $\int_C e^x \, dx + e^y \, dy$ where C is the arc of $x = y^3$ from $(-1, -1)$ to $(1, 1)$.

$$x(t) = t^3, \quad y(t) = t, \quad -1 \le t \le 1$$

$$= \int_{-1}^{1} e^{x(t)} \cdot x'(t)\, dt + e^{y(t)} \cdot y'(t)\, dt$$

$$= \int_{-1}^{1} e^{t^3} \cdot 3t^2 \, dt + \int_{-1}^{1} e^t \, dt$$

$$= e^{t^3}\Big|_{-1}^{1} + e^t \Big|_{-1}^{1}$$

$$= \left[e^{1^3} - e^{(-1)^3}\right] + \left[e^1 - e^{-1}\right]$$

$$= 2e - 2e^{-1}$$

3. Let $F(x,y) = \overbrace{-e^y \sin(x)}^{P}\mathbf{i} + \overbrace{e^y \cos(x)}^{Q}\mathbf{j}$.

(a) Find a function f such that $F = \nabla f$

$$f = \int P\,dx = \int -e^y \sin x\,dx = e^y \cos(x) + g(y)$$

$$f = \int Q\,dy = \int e^y \cos x\,dy = e^y \cos x + h(x)$$

so $f = e^y \cos(x) + C$

(b) Evaluate $\int_C F \cdot d\mathbf{r}$ where $r(t) = \langle t, t+1\rangle$, $0 \le t \le \pi$.

$$\int F \cdot dr = f\big(r(\pi)\big) - f\big(r(0)\big)$$

$* \; r(\pi) = (\pi, \pi+1)$

$r(0) = (0, 1)$

$$= f(\pi, \pi+1) - f(0,1)$$

$$= e^{\pi+1}\cos\pi - e^1 \cos 0$$

$$= -e^{\pi+1} - e$$