QUIZ 10 MATH 232, CALCULUS III, FALL 2016 NAME: SOLUTION

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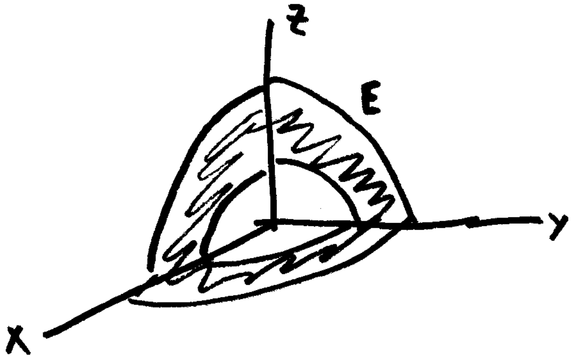
1. Evaluate $\int\int\int_E x\sqrt{x^2+y^2+z^2}\,dV$ where E is the region between $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=4$ in the first octant by first converting the integral to spherical coordinates.



$$E=\begin{cases} 1\le\rho\le 2 \\ 0\le\theta\le\pi/2 \\ 0\le\phi\le\pi/2\end{cases}$$

$$x\sqrt{x^2+y^2+z^2}=\rho\cos\theta\sin\phi\cdot\sqrt{\rho^2}$$
$$=\rho^2\cos\theta\sin\phi$$

- $$\iiint_E x\sqrt{x^2+y^2+z^2}\,dV=\int_1^2\int_0^{\pi/2}\int_0^{\pi/2}\rho^2\cos\theta\sin\phi\cdot\rho^2\sin\phi\,d\phi\,d\theta\,d\rho$$
$$=\int_1^2\rho^4 d\rho\cdot\int_0^{\pi/2}\cos\theta\,d\theta\cdot\int_0^{\pi/2}\sin^2\phi\,d\phi$$

(1) $\int_1^2\rho^4 d\rho=\frac{1}{5}\rho^5\Big|_1^2=\frac{32}{5}-\frac{1}{5}=\frac{31}{5}$

(2) $\int_0^{\pi/2}\cos\theta\,d\theta=\sin\theta\Big|_0^{\pi/2}=\sin\pi/2-\sin 0=1$

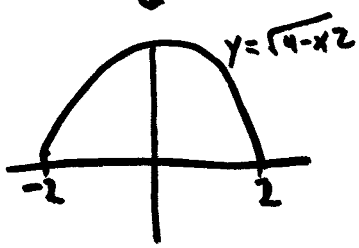
(3) $\int_0^{\pi/2}\sin^2\theta\,d\theta=\int_0^{\pi/2}\frac{1}{2}-\frac{1}{2}\cos 2\theta\,d\theta=\frac{1}{2}\theta-\frac{1}{4}\sin 2\theta\Big|_0^{\pi/2}$

$$=\left[\frac{1}{2}\left(\frac{\pi}{2}\right)-\frac{1}{4}\sin\pi\right]-\left[0-0\right]$$
$$=\frac{\pi}{4}$$

FINAL: $\frac{31}{5}\cdot 1\cdot\frac{\pi}{4}=\frac{31\pi}{20}$

2. Convert the $\int_{-2}^{2}\int_0^{\sqrt{4-x^2}}\int_0^{\sqrt{4-x^2-y^2}} xy\,dz\,dy\,dx$ to an integral in Spherical Coordinates.

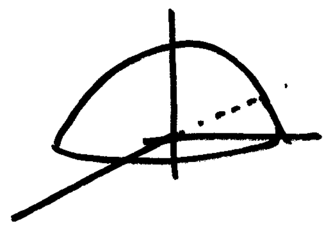
(1)



$\rightarrow \begin{cases} 0 \le \rho \le 2 \\ 0 \le \theta \le \pi \end{cases}$

(2) $z=0$ TO $z=\sqrt{4-x^2-y^2}$ IS TOP PART OF A SPHERE

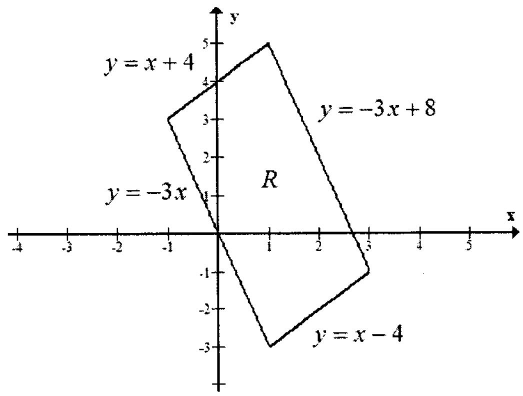
SO $0 \le \phi \le \pi/2$

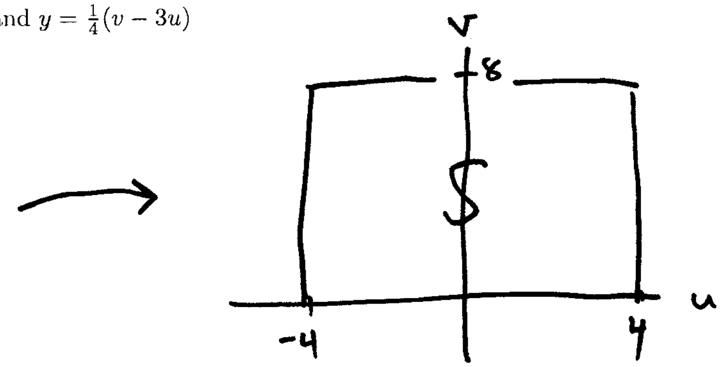


(3) $$\int_0^2\int_0^{\pi}\int_0^{\pi/2} \rho\cos\theta\sin\phi \cdot \rho\sin\theta\sin\phi \cdot \rho^2\sin\phi \, d\phi\, d\theta\, d\rho$$

$$= \int_0^2\int_0^{\pi}\int_0^{\pi/2} \rho^4\cos\theta\sin\theta \cdot \sin^3\phi \, d\phi\, d\theta\, d\rho$$

3. Evaluate $\displaystyle\int\int_R 4x + 8y\, dA$ where R is the parallelogram with vertices (-1,3), (1,-3), (3,-1), and (1,5) using the substitution $x = \frac{1}{4}(u+v)$ and $y = \frac{1}{4}(v-3u)$





(1) $y = x + 4$

$\frac{1}{4}(v-3u) = \frac{1}{4}(u+v) + 4$

$v - 3u = u + v + 16$

$-4u = 16$

$\boxed{u = -4}$

(2) $y = -3x$

$\frac{1}{4}(v-3u) = \frac{-3}{4}(u+v)$

$v - 3u = -3u - 3v$

$4v = 0$

$\boxed{v = 0}$

(3) $y = x - 4$

$\frac{1}{4}(v-3u) = \frac{1}{4}(u+v) - 4$

$v - 3u = u + v - 16$

$-4u = -16$

$\boxed{u = 4}$

(4) $y = -3x + 8$

$\frac{1}{4}(v-3u) = -\frac{3}{4}(u+v) + 8$

$v - 3u = -3u - 3v + 32$

$4v = 32$

$\boxed{v = 8}$

(5) $4x + 8y = 4 \cdot \frac{1}{4}(u+v) + 8 \cdot \frac{1}{4}(v-3u)$

$= u + v + 2v - 6u$

$= -5u + 3v$

(6) $J = \begin{vmatrix} 1/4 & 1/4 \\ -3/4 & 1/4 \end{vmatrix} = \frac{1}{16} - \frac{-3}{16} = \frac{1}{4}$

(7) $\frac{1}{4}\int_{-4}^{4}\int_{0}^{8} -5u + 3v\, dv\, du$

INSIDE: $\int_0^8 -5u + 3v\, dv = -5uv + \frac{3}{2}v^2 \Big|_0^8$

$= -40u + 96$

OUTSIDE: $\frac{1}{4}\int_{-4}^{4} -40u + 96\, du$

$= \frac{1}{4}\left[-\frac{40u^2}{2} + 96u \Big|_{-4}^{4}\right]$

$= \frac{1}{4}\left[64 - -704\right]$

$= \boxed{192}$