HW 15.1-15.2 MATH 232, CALCULUS III, FALL 2016 NAME:

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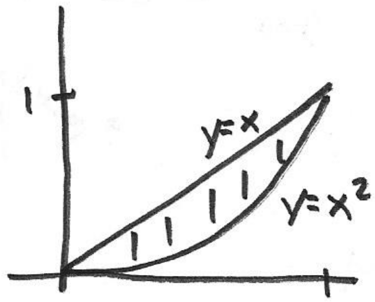
1. Evaluate the integral $\int_D\int (x^2+2y)\,dA$, over the region D where D is bounded by $y=x$ and $y=x^2$



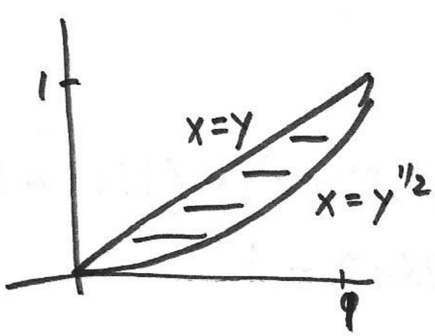
$$\{(x,y) \mid 0 \le x \le 1,\ x^2 \le y \le x\}$$

$$\int_0^1\int_{x^2}^{x} x^2+2y\,dy\,dx$$

INSIDE: $\displaystyle\int_{x^2}^{x} x^2+2y\,dy = x^2y+y^2\Big|_{x^2}^{x} = \left[x^2x+x^2\right]-\left[x^2x^2+x^4\right]$

$= -2x^4+x^3+x^2$

OUTSIDE: $\displaystyle\int_0^1 -2x^4+x^3+x^2\,dx = -\frac{2}{5}x^5+\frac{1}{4}x^4+\frac{1}{3}x^3\Big|_0^1 = \frac{11}{60}$



$$\{(x,y) \mid 0 \le y \le 1,\ y \le x \le y^{1/2}\}$$

$$\int_0^1\int_{y}^{y^{1/2}} x^2+2y\,dx\,dy$$

INSIDE: $\displaystyle\int_{y}^{y^{1/2}} x^2+2y\,dx = \frac{1}{3}x^3+2yx\Big|_{y}^{y^{1/2}} = \left[\frac{1}{3}y^{3/2}+2y^{3/2}\right]-\left[\frac{1}{3}y^3+2y^2\right]$

$= \frac{7}{3}y^{3/2}-\frac{1}{3}y^3-2y^2$

OUTSIDE: $\displaystyle\int_0^1 \frac{7}{3}y^{3/2}-\frac{1}{3}y^3-2y^2\,dy = \frac{14}{15}y^{5/2}-\frac{1}{12}y^4-\frac{2}{3}y^3\Big|_0^1 = \frac{11}{60}$

SHORTCUTS

2. Evaluate the following integrals.

(a) $\int_0^1 \int_0^2 2ye^{x+y^2}\, dx\, dy$ $= \int_0^1 \int_0^2 2ye^x e^{y^2}\, dx\, dy = \int_0^1 2ye^{y^2} dy \cdot \int_0^2 e^x dx$

- $\int_0^1 2ye^{y^2} dy \Rightarrow$ LET $u = y^2$, $du = 2y\, dy$ $\Rightarrow \int_{y=0}^{y=1} e^u du = e^u \Big|_{y=0}^{y=1}$

$= e^{y^2}\Big|_0^1 = e^1 - e^0$
$= e - 1$

- $\int_0^2 e^x dx = e^x \Big|_0^2 = e^2 - e^0 = e^2 - 1$

- FINAL: $(e-1)\cdot(e^2-1)$ OR $e^3 - e^2 - e + 1$

(b) $\int\int_R \frac{\ln x}{e^y}\, dA$ where $R = [1,2] \times [0,1]$ $= \int_1^2 \int_0^1 \frac{\ln x}{e^y}\, dy\, dx$

$= \int_1^2 \ln x\, dx \cdot \int_0^1 \frac{1}{e^y} dy$

- $\int_1^2 \ln x\, dx = x\ln x - x \Big|_{x=1}^{x=2} = (2\ln 2 - 2) - (1\ln 1 - 1)$
$= 2\ln 2 - 1$

- $\int_0^1 \frac{1}{e^y} dy = \int_0^1 e^{-y} dy = -e^{-y}\Big|_0^1 = -e^{-1} + e^0 = 1 - e^{-1}$

- FINAL: $(2\ln 2 - 1)\cdot(1 - e^{-1})$

CONSTANT

NOTE: $\int e^{x+a}\,dx = e^{x+a}$

2. Evaluate the following integrals.

(a) $\int_0^1 \int_0^2 2ye^{x+y^2}\,dx\,dy$

INSIDE: $\int_0^2 2ye^{x+y^2}\,dx = 2ye^{x+y^2}\Big|_{x=0}^{x=2} = 2ye^{2+y^2} - 2ye^{y^2}$

OUTSIDE: $\int_0^1 2ye^{2+y^2} - 2ye^{y^2}\,dy = \int_0^1 2ye^{2+y^2}\,dy - \int 2ye^{y^2}\,dy$

• $\int_0^1 2ye^{2+y^2}\,dy$ LET $u=2+y^2$, $du=2y\,dy$ $\rightarrow \int_{y=0}^{y=1} e^u\,du = e^u\Big|_{y=0}^{y=1}$

$= e^{2+y^2}\Big|_0^1 = e^3 - e^2$

• $\int_0^1 2ye^{y^2}\,dy$ LET $u=y^2$, $du=2y\,dx$ $\rightarrow \int_0^1 e^u\,du = e^u\Big|_0^1$

$= e - 1$

FINAL: $(e^3-e^2)-(e-1)$

(b) $\int\int_R \frac{\ln x}{e^y}\,dA$ where $R = [1,2] \times [0,1]$

$= \int_0^1\int_1^2 \frac{\ln x}{e^y}\,dx\,dy$

INSIDE: $\int_1^2 \frac{\ln x}{e^y}\,dx = \frac{x\ln x - x}{e^y}\Big|_{x=1}^{x=2} = \frac{2\ln 2 - 2}{e^y} - \frac{1\ln 1 - 1}{e^y}$

$= \frac{2\ln 2 - 1}{e^y}$ OR $(2\ln 2 - 1)e^{-y}$

OUTSIDE: $\int_0^1 (2\ln 2 - 1)e^{-y}\,dy = -(2\ln 2 - 1)e^{-y}\Big|_{y=0}^{y=1}$

$= -(2\ln 2 - 1)e^{-1} + (2\ln 2 - 1)e^{0}$

$= (2\ln 2 - 1)(-e^{-1} + 1)$