MATH 232 FINAL EXAM Name (print) ____________

Fall 2015 ZID ____________

Show all your work! Signature ____________

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1. **(10 points)** Let $z = \ln(\frac{x}{y})$, where $x = u^2 + 2v^2$, and $y = uv$. Find $\frac{\partial z}{\partial u}$ when $u = 1$, and $v = 1$.

2. **(15 points)** Consider the curve C given by $\mathbf{r}(t) = \cos(2t)\mathbf{i} + \sin(2t)\mathbf{j} + 3t\mathbf{k}$, $0 \leq t \leq \pi$.

(a) Find the derivative of $\mathbf{r}(t)$.

(b) Find parametric equations of the tangent line to the curve at $t = \pi/4$.

(c) Find the length of the curve.

3. **(15 points)** For each of the following, either show that the limit exists and find the limit, or show that the limit does not exist.

(a) $$\lim_{(x,y)\to(0,0)} \frac{y-x}{\sqrt{x^2+y^2}}$$

(b) $$\lim_{(x,y)\to(0,0)} \frac{2xy}{\sqrt{x^2+y^2}}.$$

4. **(15 points)** Let $P(1,0,-1), Q(1,2,3)$ and $O(0,0,0)$ be three points in $\mathbb{R}^3$.

(a) Find $\vec{OP} \times \vec{OQ}$.

(b) Find an equation of the plane passing through the points P, Q, and O.

(c) Find the area of triangle POQ.

5. **(15 points)** Let $f(x, y) = \sqrt{x^2 + 2y^2 + 1}$.

(a) Find $\nabla f(x, y)$.

(b) Find the linearization $L(x, y)$ of f at $(1, 1)$.

(c) Use the linearization in (b) to approximate $f(0.9, 1.1)$.

6. **(16 points)** Let $f(x, y) = x^2y - x^2 - 2y^2$.

(a) Find all critical points of f.

(b) Use the Second Derivatives Test to find the local maxima, local minima, and saddle points of f if there are any.

7. **(12 points)** Use the method of Lagrange multipliers to find the minimum value of $f(x, y) = xy^2$ subject to the constraint $x^2 + 2y^2 = 4$.

8. **(12 points)** Find the mass of the lamina that occupies the region $D = \{(x, y) \mid x^2 + y^2 \leq 1\}$ with the density function $\rho(x, y) = e^{-x^2-y^2}$.

9. **(10 points)** Set up a triple integral in cylindrical coordinates to represent the volume of a solid bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the sphere $z = \sqrt{8 - x^2 - y^2}$. Do not evaluate.

10. **(12 points)** Sketch the region of integration for the integral $\int_0^1 \int_{x^2}^1 \frac{\cos y}{\sqrt{y}} \, dydx$ and evaluate the integral by reversing the order of integration.

11. **(10 points)** Convert the integral $\int_0^2 \int_0^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} (x^2+y^2+z^2)^{3/2}\, dzdxdy$ into an integral in spherical coordinates. Do not evaluate.

12. **(10 points)** Set up an iterated triple integral in the order of $dzdxdy$ for the volume of the solid bounded by the coordinate planes and by $2x+y+z=6$. Do not evaluate.

13. **(15 points)** (a) Find the Jacobian $\dfrac{\partial(x,y)}{\partial(u,v)}$ of the transformation $u = x+y$, $v = y-2x$.

(b) Use your answer from (a) to re-express the integral $\int_0^1 \int_0^{1-x} (x+y)^2\ (y-2x)^3 dydx$ as an iterated integral in terms of the variables u and v. Do not evaluate.

14. **(18 points)** Consider the vector field $\mathbf{F}(x, y) = (2x \sin y)\ \mathbf{i} + (x^2 \cos y + 2y)\mathbf{j}$ in $\mathbb{R}^2$.

(a) Prove that $\mathbf{F}$ is conservative.

(b) Find a function f such that $\nabla f = \mathbf{F}$.

(c) Use (b) to evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve given by $\mathbf{r}(t) = (t+1)\mathbf{i} + t^2\mathbf{j}, 0 \leq t \leq 1$.

15. **(15 points)** Use Green's Theorem to evaluate the line integral $\oint_C xy^2dx + 2x^2ydy$, where C is the triangle from $(0,0)$ to $(1,0)$ to $(1,2)$ to $(0,0)$.