MATH 232 FINAL EXAM **Show all work for full credit** (Spring 2008)

Name (print): **Signature:**

1. (10pts) Find a Cartesian equation for $x = 5\sin t$, $y = 3\cos t$, and sketch the curve.

2. (10pts) Find the tangent line to the curve $\mathbf{r}(t) = \langle \sin t, \cos t, t \rangle$ at $(0, 1, 0)$.

3. (10pts) Find an equation of the plane through the point $(-1, 5, 4)$ and perpendicular to the line $x = 1 + 7t$, $y = t$, $z = 2 - 3t$.

4. (10pts) Find $\lim_{(x,y)\to(0,0)} \frac{x-y^2}{x+y^2}$ or show that the limit does not exist.

5. (10pts) Find the gradient and the maximum directional derivative for $f(x, y, z) = x^2yz^3$ at the point $(1, 2, 1)$.

6. (10pts) Find parametric equations of the normal line to the surface $3x^2 + y^2 - z^2 = 4$ at the point $(1, 1, 0)$.

7. (a) (5pts) Sketch the polar curve $r = 1 - \cos\theta$. (b) (10pts) Find the area of the region enclosed by the curve.

8. (15pts) Find all critical points of $f(x, y) = x^3 + y^3 - 3xy + 9$. Then use the second derivative test to determine whether each critical point is a local maximum, local minimum, or saddle point.

9. (15pts) Use Lagrange multipliers to find the maximum and minimum values of $f(x, y, z) = 6x + 6y - 4z$ subject to $x^2 + 3y^2 + z^2 = 1$.

10. (15pts) Evaluate the double integral $\int_0^2 \int_{\frac{y}{2}}^1 e^{x^2}\, dxdy$.

11. (15pts) Find the surface area for the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the circular cylinder $x^2 + y^2 = 1$.

12. (15pts) Evaluate $\int\int\int_E \sqrt{x^2 + y^2 + z^2} dV$, where E is the solid region given by $1 \leq x^2 + y^2 + z^2 \leq 4$.

13. (a) (5pts) Set up an iterated integral in the order of $dzdydx$ for the mass of the solid bounded by $z = y$, $z = 0$, and $x^2 = 1 - y$ with $x \geq 0$, assuming the density function is $\rho(x, y, z) = x$.

(b) (10pts) Evaluate the triple integral in (a).

14. (10pts) Evaluate the line integral $\int_C ydx - xdy$, where C is the positively oriented circle $x^2 + y^2 = 4$.

15. (10pts) Evaluate the line integral $\int_C (xz + 2y)ds$, where C is the line segment from $(0, 1, 0)$ to $(1, 0, 2)$.

16. (a) (5pts) State the fundamental theorem for line integrals. (b) (10pts) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = y^2\mathbf{i} + (2xy + 1)\mathbf{j} + z^2\mathbf{k}$ is a conservative vector field and C is the curve $x = \cos t$, $y = \sin t$, $z = t$, $0 \leq t \leq \pi$.