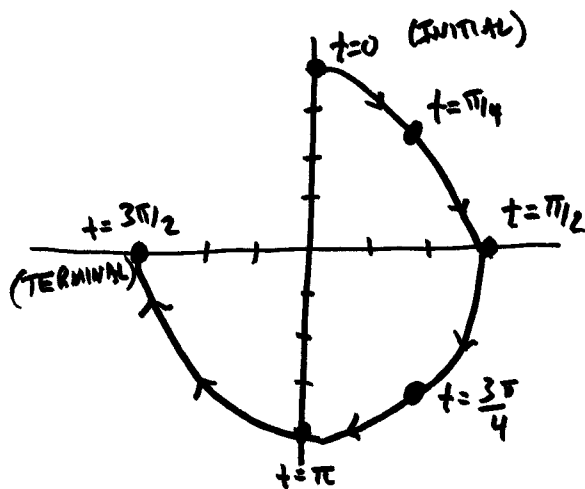


(1) CONSIDER $x = 3\sin t$, $y = 4\cos t$ for $0 \leq t \leq 3\pi/2$

(a) SKETCH



t	x	y
0	0	4
$\frac{\pi}{4}$	$\frac{3\sqrt{2}}{2}$	$\frac{4\sqrt{2}}{2}$
$\frac{\pi}{2}$	3	0
$\frac{3\pi}{4}$	$\frac{3\sqrt{2}}{2}$	$-\frac{4\sqrt{2}}{2}$
π	0	-4
$\frac{3\pi}{2}$	-3	0

(b) FIND EQUATION OF TANGENT LINE AT $t = \pi$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-4\sin t}{3\cos t} ; \quad m = \left. \frac{dy}{dx} \right|_{t=\pi} = \frac{-4\sin \pi}{3\cos \pi} = \frac{-4(0)}{3(-1)} = 0$$

TANGENT AT $t = \pi$ (OR $x = 3\sin \pi = 0$, $y = 4\cos \pi = -4$) IS

$$y + 4 = 0(x - 0)$$

$$\boxed{y = -4}$$

(c) FIND LENGTH OF CURVE: $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$

$$\Rightarrow \int_0^{3\pi/2} \sqrt{(3\cos t)^2 + (-4\sin t)^2} dt$$

(2) LET $x = \sin(\pi t)$, $y = t^2 + t$. FIND THE EQUATION OF THE TANGENT LINES) AT (0,2)

(i) POINT (0,2) OCCURS WHEN $x=0, y=2$

$$x=0 = \sin(\pi t), \quad t = 0, 1, 2, 3, 4, \dots, -1, -2, -3, \dots$$

$$y = t^2 + t \rightarrow 0 = t^2 + t - 2$$

$$\rightarrow \text{~~factored~~}$$

$$0 = (t+2)(t-1), \quad t = -2, 1$$

POINT (0,2) OCCURS WHEN $t = -2, 1$

(ii) FIND $\frac{dy}{dx}$:

$$\left. \frac{dy}{dx} \right| = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t+1}{\pi \cos(\pi t)}$$

(iii) FIND $\left. \frac{dy}{dx} \right|_{t=-2,1}$

$$\left. \frac{dy}{dx} \right|_{t=-2} = \frac{2(-2)+1}{\pi \cos(-2\pi)} = \frac{-3}{\pi} = -\frac{3}{\pi}$$

$$\left. \frac{dy}{dx} \right|_{t=1} = \frac{2(1)+1}{\pi \cos(\pi)} = \frac{3}{-\pi} = -\frac{3}{\pi}$$

(iii) TANGENT LINE(S)

$$t = -2: \quad y - 2 = -\frac{3}{\pi}(x - 0) \rightarrow y = -\frac{3}{\pi}x + 2$$

$$t = 1: \quad y - 2 = -\frac{3}{\pi}(x - 0) \rightarrow y = -\frac{3}{\pi}x + 2$$

(3) FIND THE LENGTH OF THE PARAMETRIC CURVE

$$x = t \sin(t)$$

$$y = t \cos(t)$$

$$0 \leq t \leq 1$$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

$$(i) \frac{dx}{dt} = t \cos t + \sin t$$

$$(ii) \frac{dy}{dt} = -t \sin t + \cos t$$

$$L = \int_0^1 \sqrt{(t \cos t + \sin t)^2 + (-t \sin t + \cos t)^2} dt$$

$$= \int_0^1 \sqrt{t^2 \cos^2 t + 2t \cos t \sin t + \sin^2 t + t^2 \sin^2 t - 2t \sin t \cos t + \cos^2 t} dt$$

$$= \int_0^1 \sqrt{t^2 \cos^2 t + t^2 \sin^2 t + \sin^2 t + \cos^2 t} dt$$

$$= \int_0^1 \sqrt{t^2 + 1} dt$$

TABLE OF INTEGRALS

$$= \left. \frac{t}{2} \sqrt{t^2 + 1} + \frac{1}{2} \ln |t + \sqrt{t^2 + 1}| \right|_{t=0}^{t=1}$$

$$= \left[\frac{1}{2} \sqrt{2} + \frac{1}{2} \ln |1 + \sqrt{2}| \right] - \left[0 \sqrt{1} + \frac{1}{2} \ln |0 + \sqrt{1}| \right]$$

$$= \frac{1}{2} \sqrt{2} + \frac{1}{2} \ln |1 + \sqrt{2}|$$

(4) FIND $\frac{d^2y}{dx^2}$ FOR THE PARAMETRIC CURVE

$$x = t^2 + 1$$

$$y = e^t - 1$$

(i) FIND $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{e^t}{2t}$

(ii) $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt}\left(\frac{e^t}{2t}\right)}{2t}$

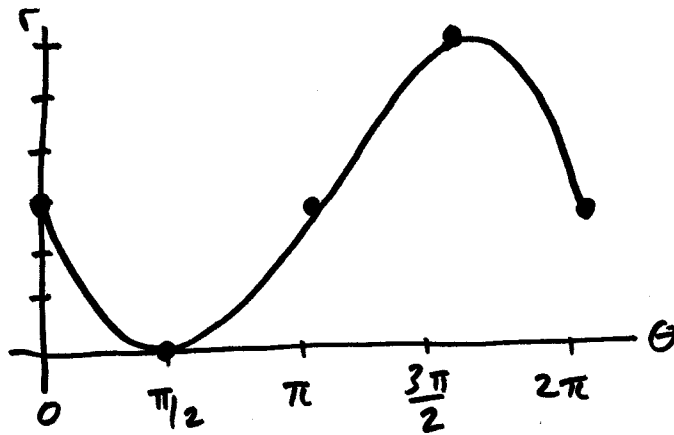
$$= \frac{(2t)(e^t) - e^t(2)}{(2t)^2}$$

$$(2t)$$

$$= \frac{2te^t - 2e^t}{(2t)^3}$$

(5) SKETCH $r = 3 - 3\sin\theta$

(i) $r = 3 - 3\sin\theta$ on $r\theta$ PLANE

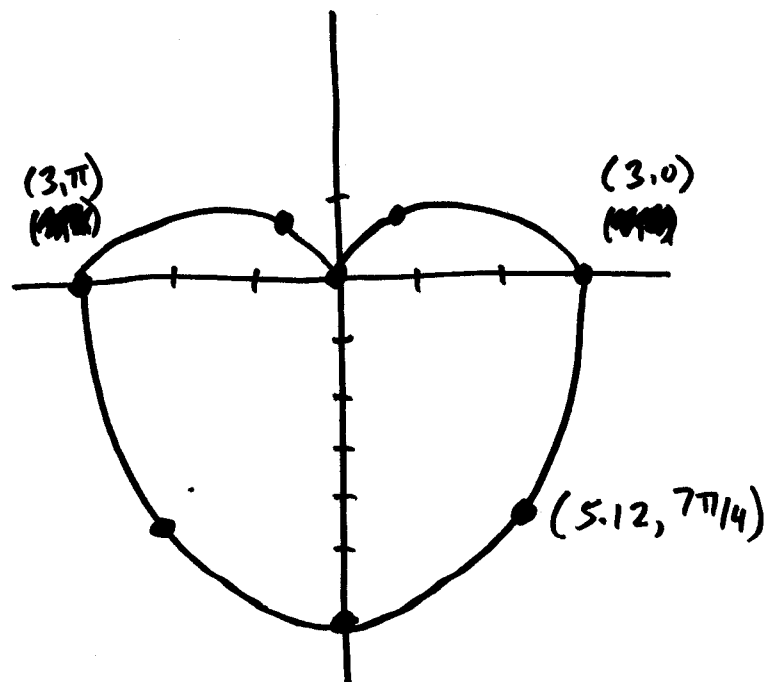


(ii) TABLE

θ	r
0	3
$\frac{\pi}{2}$	0
π	3
$\frac{3\pi}{2}$	6
2π	3

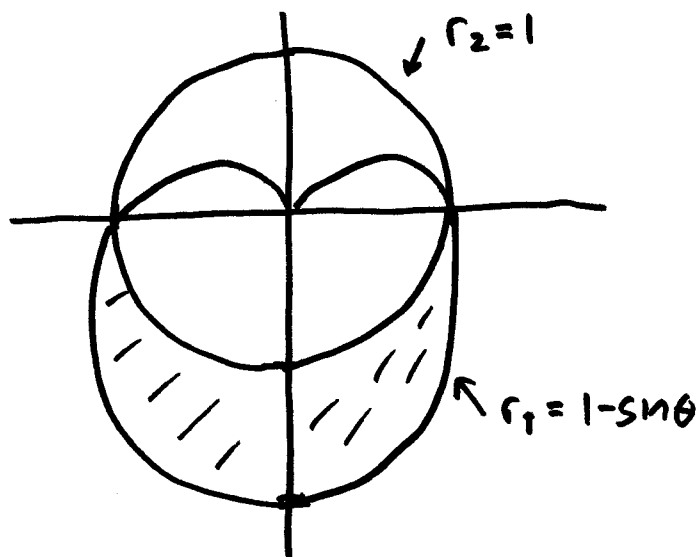
θ	r
$\frac{\pi}{4}$	$3 - 3\sin(\frac{\pi}{4}) \approx .88$
$\frac{3\pi}{4}$	$3 - 3\sin(\frac{3\pi}{4}) \approx .88$
$\frac{5\pi}{4}$	$3 - 3\sin(\frac{5\pi}{4}) \approx 5.12$
$\frac{7\pi}{4}$	$3 - 3\sin(\frac{7\pi}{4}) \approx 5.12$

(iii) SKETCH



(6) LET $r_1 = 1 - \sin \theta$, $r_2 = 1$

(a) SKETCH



(b) SETUP INTEGRAL TO FIND THE ARC LENGTH OF r_1 ON $0 \leq \theta \leq \pi/2$

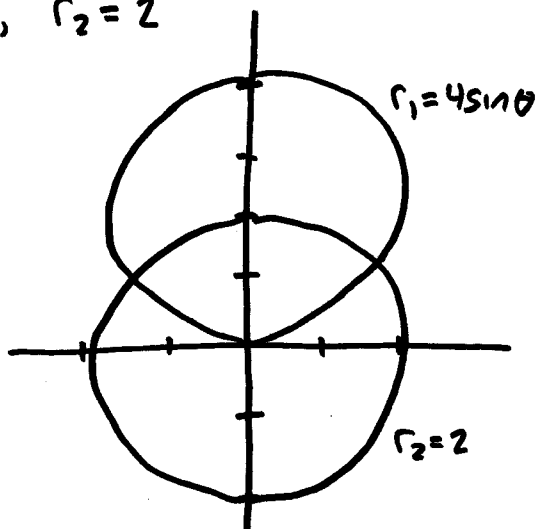
$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta = \int_0^{\pi/2} \sqrt{(1 - \sin \theta)^2 + (-\cos \theta)^2} d\theta$$

(c) SETUP INTEGRAL TO FIND AREA INSIDE r_1 , OUTSIDE r_2 .

$$\begin{aligned} & \int_{\pi}^{2\pi} \frac{1}{2} (1 - \sin \theta)^2 - \frac{1}{2} (1)^2 d\theta = \frac{1}{2} \int_{\pi}^{2\pi} (1 - \sin \theta)^2 - 1 d\theta \\ &= \frac{1}{2} \int_{\pi}^{2\pi} 1 - 2\sin \theta + \sin^2 \theta - 1 d\theta = \frac{1}{2} \int_{\pi}^{2\pi} \sin^2 \theta - 2\sin \theta d\theta \\ &= \frac{1}{2} \int_{\pi}^{2\pi} \frac{1}{2} - \frac{1}{2} \cos 2\theta - 2\sin \theta d\theta = \int_{\pi}^{2\pi} \frac{1}{4} - \frac{1}{4} \cos 2\theta - 1\sin \theta d\theta \\ &= \frac{1}{4} \theta - \frac{1}{8} \sin 2\theta + \cos \theta \Big|_{\theta=\pi}^{\theta=2\pi} \\ &= \left[\frac{1}{4} (2\pi) - \frac{1}{8} \sin 4\pi + \cos 2\pi \right] - \left[\frac{1}{4} \pi - \frac{1}{8} \sin 2\pi + \cos \pi \right] \\ &= \frac{1}{4} \pi + 2 \end{aligned}$$

(7) LET $r_1 = 4\sin\theta$, $r_2 = 2$

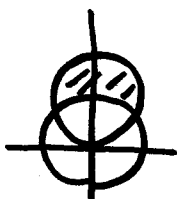
(a) SKETCH



(b) SETUP INTEGRAL TO FIND ARC LENGTH, $0 \leq \theta \leq \pi/2$ OF r_1

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta = \int_0^{\pi/2} \sqrt{(4\sin\theta)^2 + (4\cos\theta)^2} d\theta$$

(c) FIND THE AREA INSIDE r_1 AND OUTSIDE r_2



(i) FIND INTERSECTION BETWEEN r_1 AND r_2

$$\begin{aligned} 4\sin\theta &= 2 \\ \Rightarrow \sin\theta &= \frac{1}{2} \\ \Rightarrow \theta &= \pi/6, 5\pi/6 \end{aligned}$$

(ii) SETUP

$$\begin{aligned} \int_{\pi/6}^{5\pi/6} \frac{1}{2}(4\sin\theta)^2 - \frac{1}{2}(2)^2 d\theta &= \int_{\pi/6}^{5\pi/6} 8\sin^2\theta - 2 d\theta = \int_{\pi/6}^{5\pi/6} 8\left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right) - 2 d\theta \\ &= \int_{\pi/6}^{5\pi/6} 4 - 4\cos 2\theta - 2 d\theta = \int_{\pi/6}^{5\pi/6} 2 - 4\cos 2\theta d\theta \\ &= 2\theta - 2\sin 2\theta \Big|_{\theta=\pi/6}^{\theta=5\pi/6} = \left[2\left(\frac{5\pi}{6}\right) - 2\sin\left(\frac{5\pi}{3}\right)\right] - \left[2\left(\frac{\pi}{6}\right) - 2\sin\left(\frac{\pi}{3}\right)\right] \\ &= \frac{10\pi}{6} - 2\left(-\frac{\sqrt{3}}{2}\right) - \frac{2\pi}{6} + 2\left(\frac{\sqrt{3}}{2}\right) = \frac{4\pi}{3} + 2\sqrt{3} \end{aligned}$$

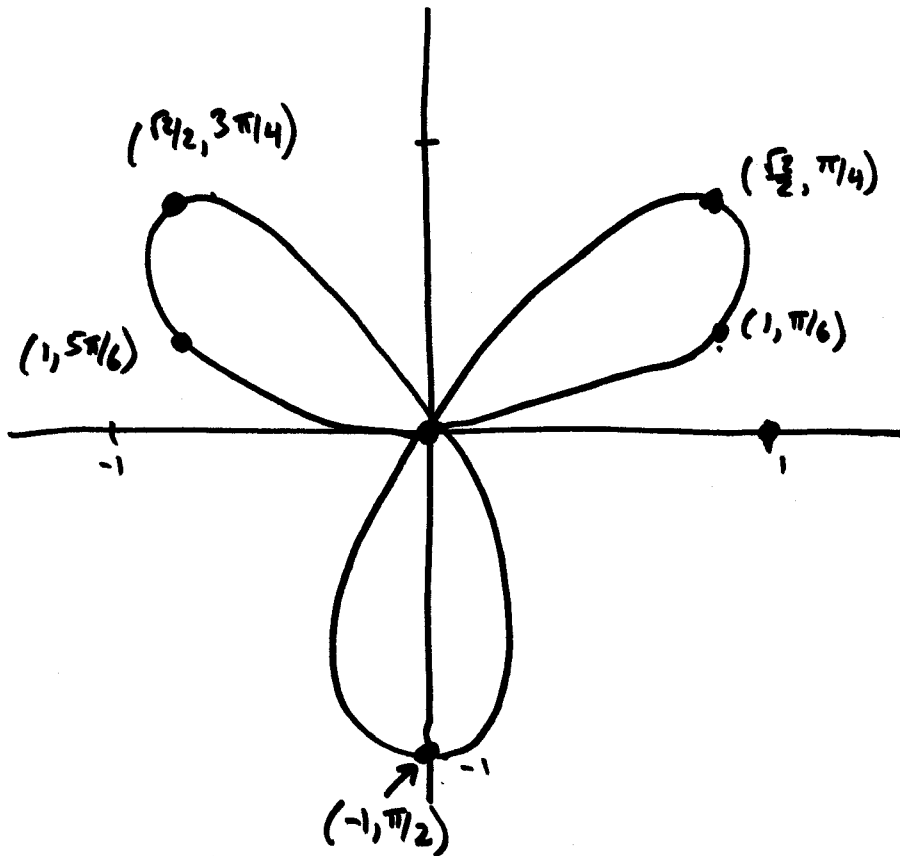
(8) CONSIDER $r = \sin 3\theta$

(a) SKETCH

θ	r
0	0
$\pi/6$	1
$\pi/4$	$\sqrt{2}/2$
$\pi/3$	0
$\pi/2$	-1

θ	r
$2\pi/3$	0
$3\pi/4$	$\sqrt{2}/2$
$5\pi/6$	1
π	0

POINTS REPEAT



(8b) EVALUATE SLOPE OF THE TANGENT LINE AT $\theta = \pi/6$

$$\begin{aligned}\frac{dy}{dx} &= \frac{\frac{dr}{d\theta} \sin\theta + r \cos\theta}{\frac{dr}{d\theta} \cos\theta - r \sin\theta} \\ &= \frac{(3 \cos 3\theta) \sin\theta + \sin 3\theta \cos\theta}{(3 \cos 3\theta) \cos\theta - \sin 3\theta \sin\theta}\end{aligned}$$

$$\begin{aligned}\left. \frac{dy}{dx} \right|_{\theta=\pi/6} &= \frac{3 \cos(\pi/2) \sin(\pi/6) + \sin(\pi/2) \cos(\pi/6)}{3 \cos(\pi/2) \cos(\pi/6) - \sin(\pi/2) \sin(\pi/6)} \\ &= \frac{\sqrt{3}/2}{-1/2} = -\sqrt{3}\end{aligned}$$

(8c) TANGENT LINE?

(i) SLOPE: $m = -\sqrt{3}$

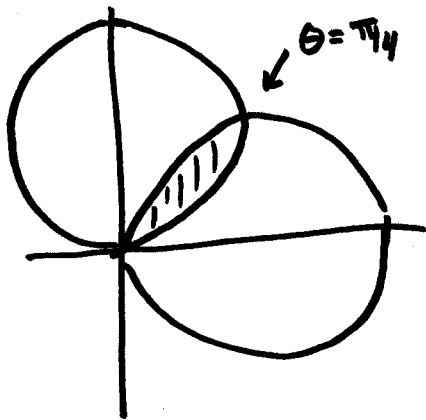
(ii) POINT: $x = r \cos\theta = \sin 3\theta \cos\theta$, $x = \sin \pi/2 \cos \pi/6 = \sqrt{3}/2$
 $y = r \sin\theta = \sin 3\theta \sin\theta$, $y = \sin \pi/2 \sin \pi/6 = 1/2$

$$y - \frac{1}{2} = -\sqrt{3} \left(x - \frac{\sqrt{3}}{2} \right)$$

$$y - \frac{1}{2} = -\sqrt{3}x + \frac{3}{2}$$

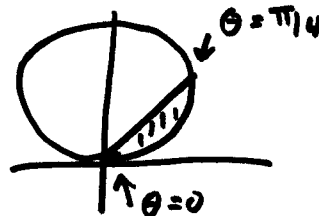
$$y = -\sqrt{3}x + 2$$

(9) FIND AREA OF THE REGION INSIDE $r = \cos \theta$, $r = \sin \theta$



(i) $\cos \theta = \sin \theta \rightarrow \theta = \pi/4$

(ii) $A = 2 \cdot \int_0^{\pi/4} \frac{1}{2} (\sin \theta)^2 d\theta$



THIS IS HALF
THE AREA

$$A = 2 \cdot \int_0^{\pi/4} \frac{1}{2} (\sin \theta)^2 d\theta = \int_0^{\pi/4} \sin^2 \theta d\theta$$

$$= \int_0^{\pi/4} \frac{1}{2} - \frac{1}{2} \cos 2\theta d\theta$$

$$= \frac{1}{2} \theta - \frac{1}{4} \sin 2\theta \Big|_{\theta=0}^{\theta=\pi/4}$$

$$= \left[\frac{1}{2} \left(\frac{\pi}{4} \right) - \frac{1}{4} \sin \left(\frac{\pi}{2} \right) \right] - \left[\frac{1}{2} (0) - \frac{1}{4} \sin 0 \right]$$

$$= \frac{\pi}{8} - \frac{1}{4}$$

(10) DETERMINE WHETHER VECTORS ARE PARALLEL, ORTHOGONAL, OR NEITHER.

(a) $\vec{a} = \langle -3, 5, 11 \rangle$, $\vec{b} = \langle 10, -5, 5 \rangle$

ORTHOGONAL? CHECK IF $\vec{a} \cdot \vec{b} = 0$

$$\langle -3, 5, 11 \rangle \cdot \langle 10, -5, 5 \rangle = -30 - 25 + 55 = 0 \quad \checkmark$$

VECTORS ARE ORTHOGONAL

EVEN THOUGH PROBLEM IS COMPLETE, LET'S CHECK FOR PARALLEL

(i) FIRST (AND EASIER) WAY. DOES $\vec{a} = \lambda \vec{b}$, WHERE λ IS CONSTANT?

$$-3 = \lambda(10) \rightarrow \lambda = -3/10$$

$$5 = \lambda(-5) \rightarrow \lambda = -1$$

$$11 = \lambda(5) \rightarrow \lambda = 11/5$$

THEY NEED TO MATCH. TO BE PARALLEL.

(ii) SECOND (AND HARDER) WAY. DOES $\vec{a} \times \vec{b} = 0$

$$\begin{aligned} \vec{a} \times \vec{b} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 5 & 11 \\ 10 & -5 & 5 \end{vmatrix} = \vec{i} \begin{vmatrix} 5 & 11 \\ -5 & 5 \end{vmatrix} - \vec{j} \begin{vmatrix} -3 & 11 \\ 10 & 5 \end{vmatrix} + \vec{k} \begin{vmatrix} -3 & 5 \\ 10 & -5 \end{vmatrix} \\ &= \vec{i}(25 + 55) - \vec{j}(-15 - 110) + \vec{k}(15 - 50) \\ &= 80\vec{i} + 125\vec{j} - 35\vec{k} \end{aligned}$$

$$\neq 0$$

$$(106) \quad \vec{a} = \langle 1, 2, -1 \rangle, \quad \vec{b} = \langle 3, 6, 3 \rangle$$

(i) ORTHOGONAL? $\vec{a} \cdot \vec{b} = 0?$

$$\langle 1, 2, -1 \rangle \cdot \langle 3, 6, 3 \rangle = 3 + 12 + -3 = 12 \neq 0$$

SO NOT ORTHOGONAL.

(ii) PARALLEL?

• EASY WAY. DOES $\vec{a} = \lambda \vec{b}$?

$$1 = \lambda(3) \rightarrow \lambda = 1/3$$

$$2 = \lambda(6) \rightarrow \lambda = 1/3$$

$$-1 = \lambda(3) \rightarrow \lambda = -1/3$$

↓ NOT THE SAME. NOT PARALLEL

• HARD WAY.

$$\begin{aligned} \vec{a} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & 6 & 3 \end{vmatrix} = \hat{i} \begin{vmatrix} 2 & -1 \\ 6 & 3 \end{vmatrix} - \hat{j} \begin{vmatrix} 1 & -1 \\ 3 & 3 \end{vmatrix} + \hat{k} \begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix} \\ &= \hat{i}(6+6) - \hat{j}(3+3) + \hat{k}(6-6) \\ &= 12\hat{i} - 6\hat{j} + 0\hat{k} \end{aligned}$$

(iii) ANSWER IS NEITHER. ALSO CALLED SKEWED

$$(10c) \quad \vec{a} = \langle 2, -3, 8 \rangle, \quad \vec{b} = \langle -6, 9, -24 \rangle$$

(i) ORTHOGONAL?

$$\vec{a} \cdot \vec{b} = \langle 2, -3, 8 \rangle \cdot \langle -6, 9, -24 \rangle = -12 - 27 - 192 = -231 \neq 0$$

NOT ORTHOGONAL

(iii) PARALLEL?

EASY WAY. DOES $\vec{a} = \lambda \vec{b}$?

$$2 = \lambda(-6) \rightarrow \lambda = -1/3$$

$$-3 = \lambda(9) \rightarrow \lambda = -1/3$$

$$8 = \lambda(-24) \rightarrow \lambda = -1/3$$

ALL MATCH. \vec{a} AND \vec{b}
ARE PARALLEL.

HARD WAY DOES $\vec{a} \times \vec{b} = 0$

$$\begin{aligned} \vec{a} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 8 \\ -6 & 9 & -24 \end{vmatrix} = \hat{i} \begin{vmatrix} -3 & 8 \\ 9 & -24 \end{vmatrix} - \hat{j} \begin{vmatrix} 2 & 8 \\ -6 & -24 \end{vmatrix} + \hat{k} \begin{vmatrix} 2 & -3 \\ -6 & 9 \end{vmatrix} \\ &= \hat{i}(72 - 72) - \hat{j}(-48 + 48) + \hat{k}(18 - 18) \\ &= 0\hat{i} + 0\hat{j} + 0\hat{k} \\ &= 0 \end{aligned}$$

SINCE $\vec{a} \times \vec{b} = 0$, \vec{a} AND \vec{b} ARE PARALLEL.

(11) GIVEN THAT $\vec{a} = \langle 3, -4, -5 \rangle$ AND $\vec{b} = \langle -2, 1, 2 \rangle$ AND $\vec{c} = \langle -6, -3, 2 \rangle$.

(a) EVALUATE $2\vec{a} - 3\vec{b} + 2\vec{c}$

$$\begin{aligned} &= 2\langle 3, -4, -5 \rangle - 3\langle -2, 1, 2 \rangle + 2\langle -6, -3, 2 \rangle \\ &= \langle 6, -8, -10 \rangle + \langle 6, -3, -6 \rangle + \langle -12, -6, 4 \rangle \\ &= \langle 0, -17, -12 \rangle \end{aligned}$$

(b) EVALUATE $\frac{|\vec{a}| |\vec{b}|}{|\vec{c}|}$

(i) $|\vec{a}| = \sqrt{(3)^2 + (-4)^2 + (-5)^2} = \sqrt{50}$

(ii) $|\vec{c}| = \sqrt{(-6)^2 + (-3)^2 + (2)^2} = \sqrt{49} = 7$

(iii) $\frac{|\vec{a}| |\vec{b}|}{|\vec{c}|} = \frac{\sqrt{50} \langle -2, 1, 2 \rangle}{\sqrt{49}} = \frac{5\sqrt{2}}{7} \langle -2, 1, 2 \rangle$
 $= \left\langle -\frac{10\sqrt{2}}{7}, \frac{5\sqrt{2}}{7}, \frac{10\sqrt{2}}{7} \right\rangle$

(c) FIND ANGLE BETWEEN \vec{b} AND \vec{c}

USE $\vec{b} \cdot \vec{c} = |\vec{b}| |\vec{c}| \cos \theta \rightarrow \cos \theta = \frac{\vec{b} \cdot \vec{c}}{|\vec{b}| |\vec{c}|}$

• $\vec{b} \cdot \vec{c} = \langle -2, 1, 2 \rangle \cdot \langle -6, -3, 2 \rangle = 12 - 3 + 4 = 13$

• $|\vec{b}| = \sqrt{(-2)^2 + (1)^2 + (2)^2} = \sqrt{9} = 3$, $|\vec{c}| = 7$

• $\cos \theta = \frac{13}{3 \cdot 7} = \frac{13}{21} \rightarrow \theta = \cos^{-1}\left(\frac{13}{21}\right) \approx \text{~~52~~ } 52^\circ$

(11d) ARE \vec{b} AND \vec{a} ORTHOGONAL?

$$\vec{a} \cdot \vec{b} = \langle 3, -4, -5 \rangle \cdot \langle -2, 1, 2 \rangle = -6 - 4 - 10 = -20$$

SINCE $\vec{a} \cdot \vec{b} \neq 0$, \vec{a} AND \vec{b} ARE NOT ORTHOGONAL

(11e) FIND $\vec{b} \times \vec{c}$

$$\begin{aligned}\vec{b} \times \vec{c} &= \begin{vmatrix} i & j & k \\ -2 & 1 & 2 \\ -6 & -3 & 2 \end{vmatrix} = i \begin{vmatrix} 1 & 2 \\ -3 & 2 \end{vmatrix} - j \begin{vmatrix} -2 & 2 \\ -6 & 2 \end{vmatrix} + k \begin{vmatrix} -2 & 1 \\ -6 & -3 \end{vmatrix} \\ &= i(2+6) - j(-4+12) + k(6+6) \\ &= 8i - 8j + 12k\end{aligned}$$

$$(11f) |\vec{b} \times \vec{c}| = \sqrt{(8)^2 + (-8)^2 + 12^2} = \sqrt{272}$$

(11g) FIND THE VECTOR PROJECTION OF \vec{b} ONTO \vec{c}

$$\text{proj}_{\vec{c}} \vec{b} = \left(\frac{\vec{c} \cdot \vec{b}}{|\vec{c}|^2} \right) \vec{c} = \left(\frac{13}{7^2} \right) \langle -6, -3, 2 \rangle = \left\langle \frac{-78}{49}, \frac{-39}{49}, \frac{26}{49} \right\rangle$$

(11h) FIND DIRECTION COSINES AND ANGLES OF \vec{a}

$$\cos \alpha = \frac{3}{\sqrt{50}}, \quad \cos \beta = \frac{-4}{\sqrt{50}}, \quad \cos \gamma = \frac{-5}{\sqrt{50}}$$

$$\alpha = 65^\circ, \quad \beta = 124^\circ, \quad \gamma = 135^\circ$$

(12) GIVEN THREE POINTS $P(2, 1, 2)$, $Q(3, 8, -6)$, AND $R(-2, -3, 1)$.
FIND

(a) THE VECTOR EQUATION OF THE LINE PASSING THROUGH
 $P(2, 1, 2)$ AND $Q(3, 8, -6)$

(i) DIRECTION VECTOR: $\vec{PQ} = \langle 3-2, 8-1, -6-2 \rangle = \langle 1, 7, -8 \rangle$

(ii) POINT/VECTOR: $\langle 2, 1, 2 \rangle$

$$r(t) = \langle 2, 1, 2 \rangle + t \langle 1, 7, -8 \rangle$$

(b) THE PARAMETRIC EQUATIONS OF THE LINE PASSING THROUGH
 $P(2, 1, 2)$ AND $R(-2, -3, 1)$

(i) DIRECTION VECTOR: $\vec{PR} = \langle -2-2, -3-1, 1-2 \rangle = \langle -4, -4, -1 \rangle$

(ii) POINT/VECTOR: $\langle 2, 1, 2 \rangle$

$$x = 2 - 4t, \quad y = 1 - 4t, \quad z = 2 - t$$

(c) ARE \vec{PQ} AND \vec{PR} ORTHOGONAL, PARALLEL, OR NEITHER

DOT: $\vec{PQ} \cdot \vec{PR} = \langle 1, 7, -8 \rangle \cdot \langle -4, -4, -1 \rangle = -4 - 28 + 8 = -24 \neq 0$ (NOT ORTHOGONAL)

CROSS: $\vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ 1 & 7 & -8 \\ -4 & -4 & -1 \end{vmatrix} = i \begin{vmatrix} 7 & -8 \\ -4 & -1 \end{vmatrix} - j \begin{vmatrix} 1 & -8 \\ -4 & -1 \end{vmatrix} + k \begin{vmatrix} 1 & 7 \\ -4 & -4 \end{vmatrix}$

$$= i(-7-32) - j(-1-32) + k(-4+28)$$

$$= -39i + 33j + 24k \neq 0$$

NOT PARALLEL

ANSWER: NEITHER

(2d) FIND EQUATION OF THE PLANE THAT PASSES THROUGH
 $P(2, 1, 2)$, $Q(3, 8, -6)$, AND $R(-2, -3, 1)$.

(i) NEED A POINT: ANY OF THE THREE WILL WORK

$$P(2, 1, 2)$$

(ii) NEED A NORMAL VECTOR (A VECTOR PERPENDICULAR TO
THE PLANE)

• SINCE \vec{PQ} AND \vec{PR} LIE ON THE PLANE, THEIR
CROSS PRODUCT WILL BE PERPENDICULAR TO THE
PLANE.

EQUATION OF PLANE:

$$-39(x-2) + 33(y-1) + 24(z-2) = 0$$