Name:

Show all work for full credit.

Here is a list of some identities and formulas.

$$\sin^2(u) = \frac{1}{2}\left(1 - \cos(2u)\right)$$

$$\cos^2(u) = \frac{1}{2}\left(1 + \cos(2u)\right)$$

$$2\sin(u)\cos(u) = \sin(2u)$$

$$\text{proj}_{\vec{a}}\vec{b} = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\right)\vec{a}$$

$$\text{comp}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$$

| $\theta$ | $\cos(\theta)$ | $\sin(\theta)$ |
|---|---|---|
| $-\pi$ | -1 | 0 |
| $-5\pi/6$ | $-\sqrt{3}/2$ | -1/2 |
| $-3\pi/4$ | $-\sqrt{2}/2$ | -$\sqrt{2}/2$ |
| $-2\pi/3$ | $-1/2$ | $-\sqrt{3}/2$ |
| $-\pi/2$ | 0 | -1 |
| $-\pi/3$ | $1/2$ | $-\sqrt{3}/2$ |
| $-\pi/4$ | $\sqrt{2}/2$ | $-\sqrt{2}/2$ |
| $-\pi/6$ | $\sqrt{3}/2$ | -1/2 |
| 0 | 1 | 0 |
| $\pi/6$ | $\sqrt{3}/2$ | $1/2$ |
| $\pi/4$ | $\sqrt{2}/2$ | $\sqrt{2}/2$ |
| $\pi/3$ | $1/2$ | $\sqrt{3}/2$ |
| $\pi/2$ | 0 | 1 |
| $2\pi/3$ | $-1/2$ | $\sqrt{3}/2$ |
| $3\pi/4$ | $-\sqrt{2}/2$ | $\sqrt{2}/2$ |
| $5\pi/6$ | $-\sqrt{3}/2$ | $1/2$ |
| $\pi$ | -1 | 0 |

1. Consider the curve $x = 3\sin t$, $y = 4\cos t$ for $0 \le t \le \dfrac{3\pi}{2}$.

   (a) Graph the curve, indicating initial and terminal points, and the direction of the curve.

   (b) Find the equation of the tangent line at the point where $t = \pi$.

   (c) Set up the equation (do not evaluate) indicating the length of the given curve.

2. Let $x = \sin(\pi t)$ and $y = t^2 + t$. Find the equation of the tangent lines to the curve at $(0, 2)$.

3. Find the length of the parametric curve

$$\begin{aligned} x &= t\sin(t) \\ y &= t\cos(t) \\ 0 &\le t \le 1 \end{aligned}$$

4. Find $\frac{d^2y}{dx^2}$ for the parametric curve

$$\begin{aligned} x &= t^2 + 1 \\ y &= e^t - 1 \end{aligned}$$

5. Sketch $r = 3 - 3\sin(\theta)$.

6. Let $r_1 = 1 - \sin(\theta)$ and $r_2 = 1$.

   (a) Sketch the given functions on the same graph.

   (b) Set up an integral to find the arc length of $r_1$ when $0 \le \theta \le \pi/2$. Do not evaluate.

   (c) Set up an integral to find the area inside $r_1$ and outside $r_2$. Evaluate to find the area.

7. Let $r_1 = 4\sin(\theta)$ and $r_2 = 2$.

   (a) Sketch the given functions on the same graph.

   (b) Set up an integral to find the arc length of $r_1$ when $0 \le \theta \le \pi/2$. Do not evaluate.

   (c) Set up an integral to find the area inside $r_1$ and outside $r_2$. Evaluate to find the area.

8. Consider the curve $r = \sin(3\theta)$.

   (a) Sketch the curve $r$.

   (b) Evaluate the slope of the tangent line at $\theta = \pi/6$.

   (c) Find the equation of the tangent line at $\theta = \pi/6$.

9. Find the area of the region inside both the curve $r = \cos\theta$ and $r = \sin\theta$.

10. Determine whether the vectors are parallel, orthogonal, or neither.

    (a) $\vec{a} = \langle -3, 5, 11\rangle$, $\vec{b} = \langle 10, -5, 5\rangle$

    (b) $\vec{a} = i + 2j - k$, $\vec{b} = 3i + 6j + 3k$

(c) $\vec{a} = \langle 2, -3, 8 \rangle$ and $\vec{b} = \langle -6, 9, -24 \rangle$.

11. Given that $\vec{a} = \langle 3, -4, -5 \rangle$, $\vec{b} = \langle -2, 1, 2 \rangle$, and $\vec{c} = \langle -6, -3, 2 \rangle$. Answer the following questions.

    (a) Evaluate $2\vec{a} - 3\vec{b} + 2\vec{c}$

    (b) Evaluate $\dfrac{|\vec{a}|\vec{b}}{|\vec{c}|}$

    (c) Find the angle between $\vec{b}$ and $\vec{c}$.

    (d) Are the vectors $\vec{a}$ and $\vec{b}$ orthogonal to each other? Why or why not.

    (e) Find $\vec{b} \times \vec{c}$

    (f) Find $|\vec{b} \times \vec{c}|$

    (g) Find the vector projection of $\vec{b}$ onto $\vec{c}$.

    (h) Find the direction cosines and the direction angles of the vector $\vec{a}$. Round your answer to nearest degrees.

12. Given three points $P(2, 1, 2)$, $Q(3, 8, -6)$, and $R(-2, -3, 1)$, find

    (a) Find the **vector** equation of the line that passes through the points $P(2, 1, 2)$ and $Q(3, 8, -6)$.

    (b) Find the **parametric** equations of the line that passes through the points $P(2, 1, 2)$ and $R(-2, -3, 1)$.

    (c) Are the lines $PQ$ and $PR$ parallel, orthogonal, or neither.

    (d) Find the equation of the plane that passes through the points $P(2, 1, 2)$, $Q(3, 8, -6)$, and $R(-2, -3, 1)$.