Math 232

Calculus III

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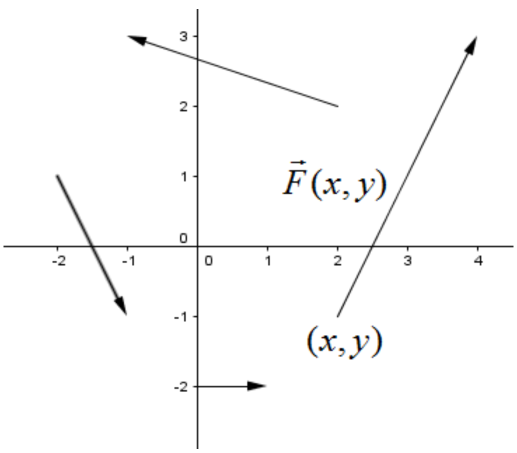
16.1 Vector Fields

Definition 1: Vector Fields

Let D be a set in $\mathbf{R}^2$. A Vector Field on $\mathbf{R}^2$ is a function F that assigns each point (x, y) a two dimensional vector $F(x, y)$.

We write $F(x, y) = P(x, y)i + Q(x, y)j = \langle P(x, y), Q(x, y) \rangle$.



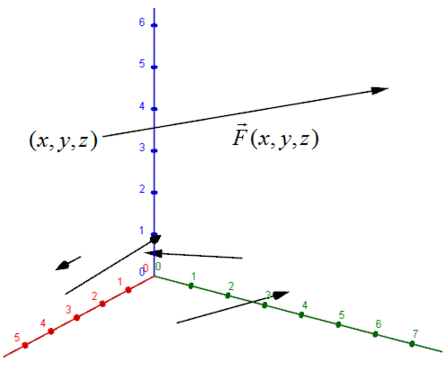
We write

$$F(x, y) = P(x, y)i + Q(x, y)j$$

$$= \langle P(x, y), Q(x, y) \rangle$$

Definition 2: Vector Field on $\mathbf{R}^3$

Let E be a subset of $\mathbf{R}^3$. A vector field is a function F that assigns to each point (x, y, z) in E a three-dimensional vector $F(x, y, z)$.



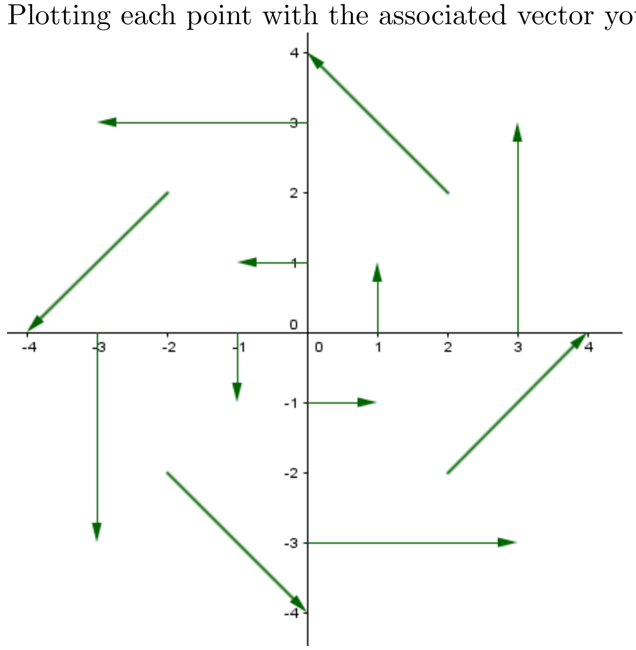
Example 1

Sketch the vector field of $F(x, y) = -yi + xj = \langle -y, x \rangle$.

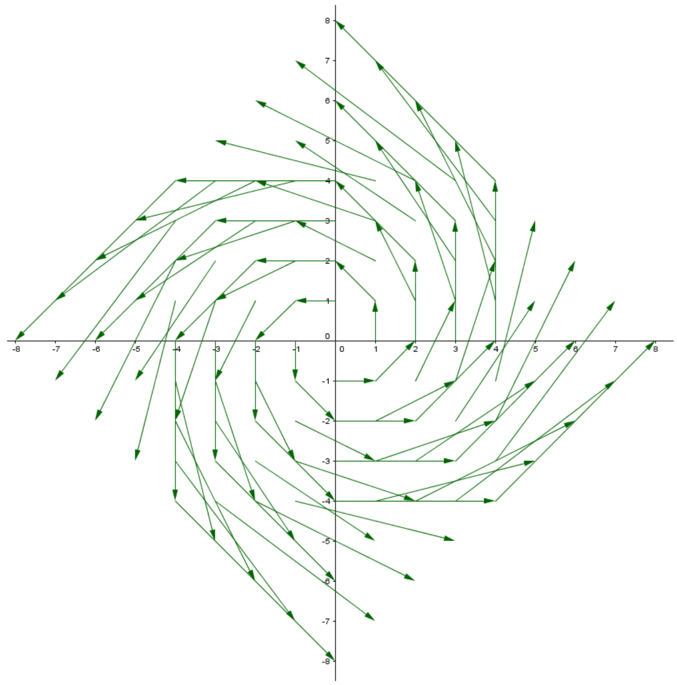
Without the aid of a computer the best to sketch these is the same as how you learned to sketch functions in algebra. Choose many points (x, y) and sketch the associated vector. These can be time consuming since you need to sketch usually around 20 to get a good idea of what the vector field looks like.

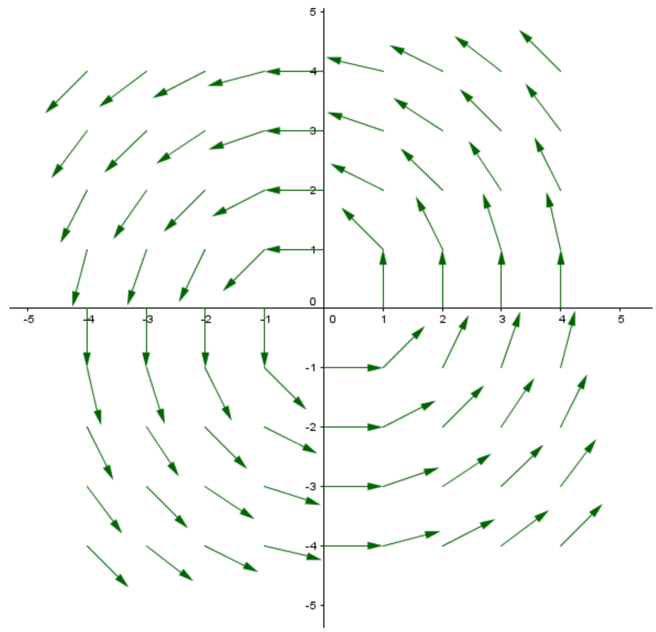
(x, y)	$F(x, y)$	(x, y)	$F(x, y)$
$(1, 0)$	$< 0, 1 >$	(-1,0)	$< 0, -1 >$
$(2, 2)$	$< -2, 2 >$	(-2,-2)	$< 2, -2 >$
$(3, 0)$	$< 0, 3 >$	$(-3, 0)$	$< 0, -3 >$
$(0, 1)$	$< -1, 0 >$	$(0, -1)$	$< 1, 0 >$
$(-2, -2)$	$< -2, -2 >$	$(2, -2)$	$< 2, 2 >$
$(0, 3)$	$< -3, 0 >$	(0,3)	$< 3, 0 >$

Plotting each point with the associated vector you get



If you use a computer to generate the vector field you get two types of fields. The first one is exactly what you get if you plot the points with its associated vector. The second is when you scale the vector down so it's not so messy. In the textbook homework problems, they will look like the second one.





Most of the homework consists of matching a function to its associated vector field. The easiest way is to write down some points with their associated vectors and see which vector field matches.

Definition 3: Gradient Fields

Given a vector function $f(x, y)$ and its gradient $\nabla f(x, y) = \langle f_x(x, y), f_y(x, y)\rangle$, a gradient vector field is the vector field using ∇f.

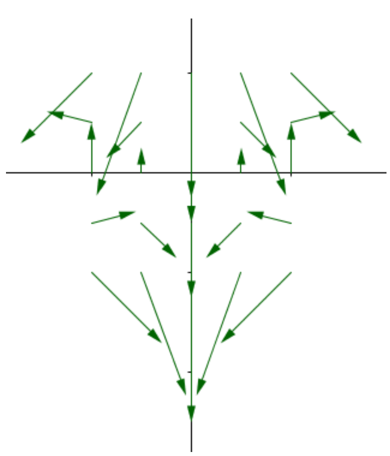
Example 2

Find the gradient vector field of $f(x, y) = x^2y - y^3$.

1. First, let's find the gradient vector function ∇f.

$$\nabla f(x, y) = \langle 2xy, x^2 - 3y^2\rangle$$

2. Plot many points. I'll just write out a couple. Note that the vectors are scaled back.



(x, y)	$\langle 2xy, x^2 - 3y^2\rangle$
$(1, 1)$	$< 2, -2 >$
$(-1, -1)$	$< 2, -2 >$
$(2, 0)$	$< 0, 4 >$
$(0, -1)$	$< 0, -3 >$