Math 232

Calculus III

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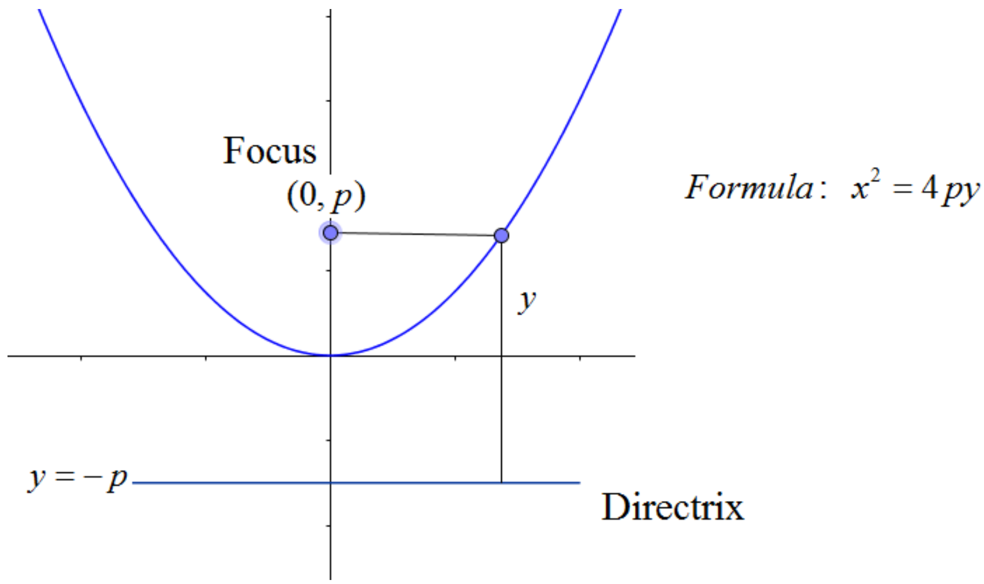
10.5 Conic Sections

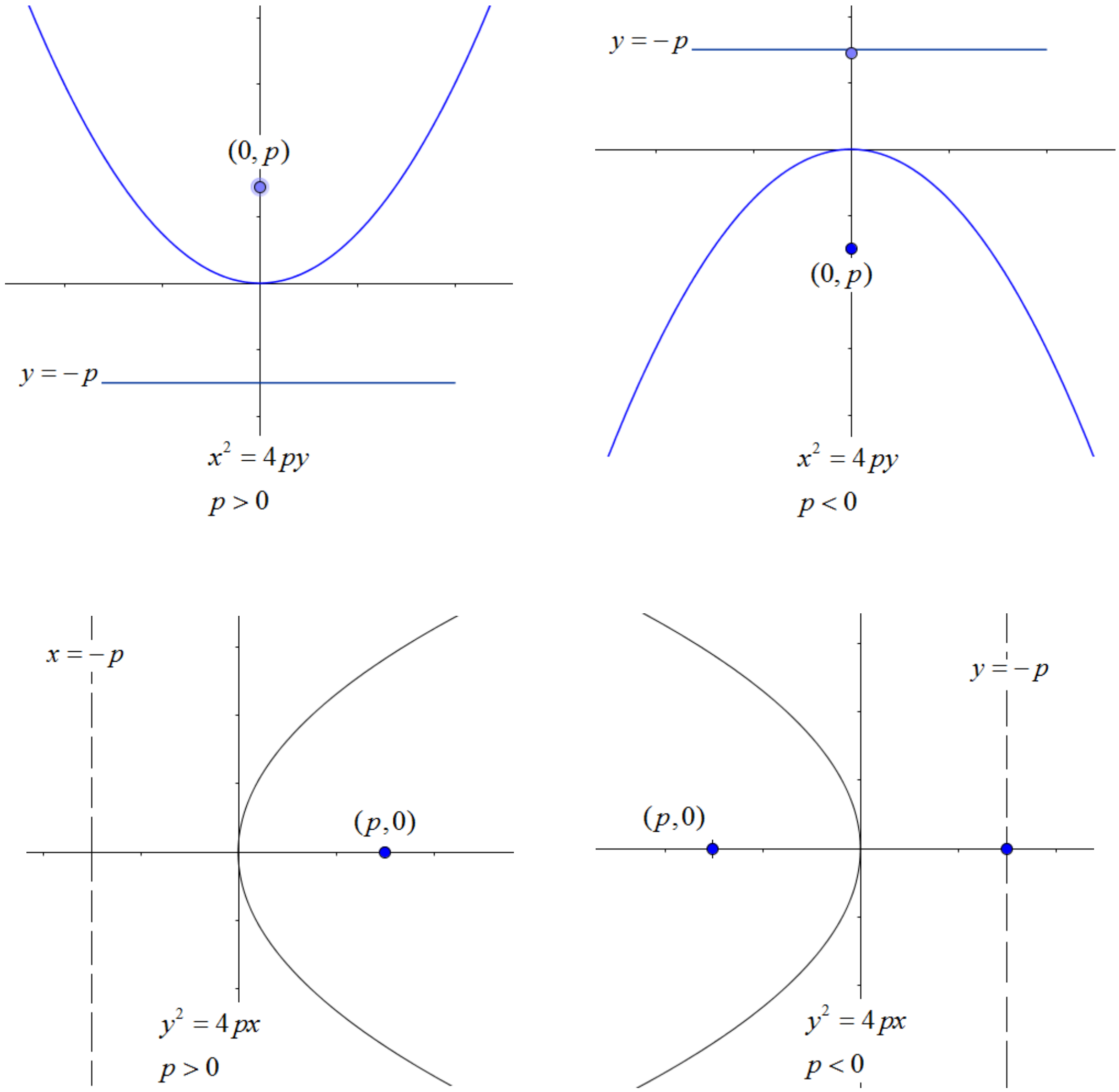
Definition 1: Parabola

A set of points in a plane that are equidistance from a fixed point F (called the focus) and a straight line (called the directrix).



Note: You are used to using $y = ax^2$ where $a = \dfrac{1}{4p}$

There are four forms for a parabola. They are shown below



Example 1

Let $3x^2 = -8py$. Find the vertex, focus, directrix and sketch.

1. First thing let's set it up in the correct form: $x^2 = \dfrac{-8}{3}y$

2. The general form is $x^2 = 4py$. Find p by solving

$$4p = -\frac{8}{3}$$

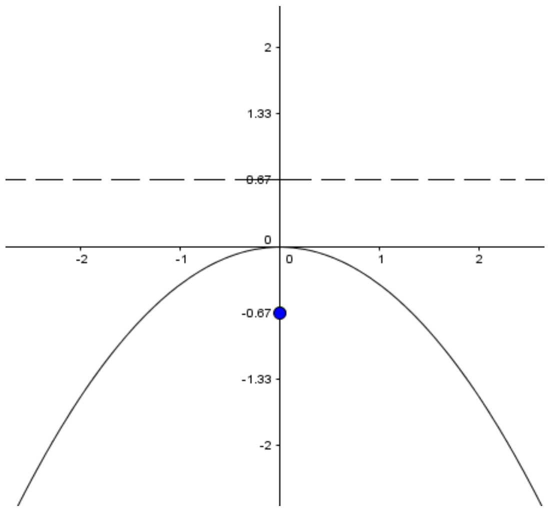
$$p = -\frac{2}{3}$$

3. Focus: $(0, -2/3)$

4. Vertex: $(0, 0)$

5. Directrix: $y = 2/3$

6. Sketch



Example 2

Sketch $(y-2)^2 = 2x + 1$.

We can ignore the transformation on the function and focus just on the general form:

$$y^2 = 2x$$

1. Find p

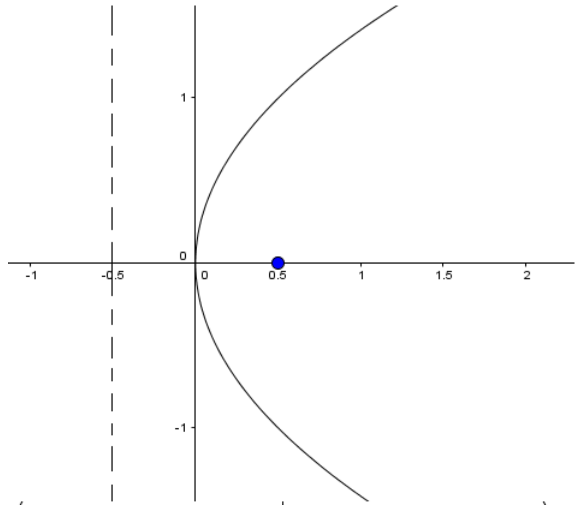
$$4p = 2$$
$$p = \frac{1}{2}$$

2. Focus: $(1/2, 0)$

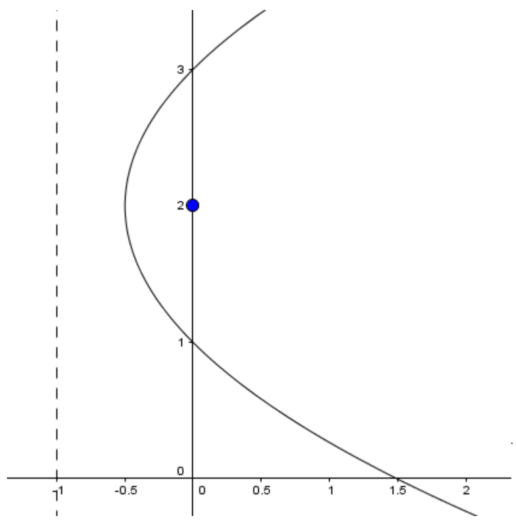
3. Directrix: $x = -1/2$

4. Vertex: $(0, 0)$.

$y^2 = 2x$ looks like



But this is not our function. We need to do a tranformation on it. The $(y-2)^2$ part tells us to shift up 2 units. The $2x+1 = 2(x+1/2)$ tells us to shift left $1/2$ units. This gives us

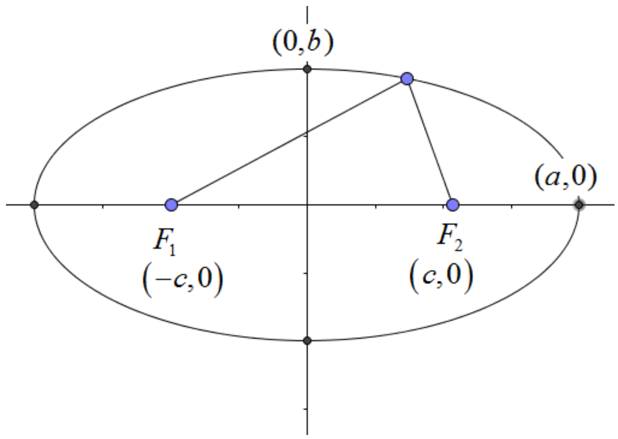


with focus $(0,1)$ and directrix $x = -1$.

Definition 2: Ellipse

A set of points on a plane such that the sum of whose distances from two fixed points F_1 and F_2 is constant.

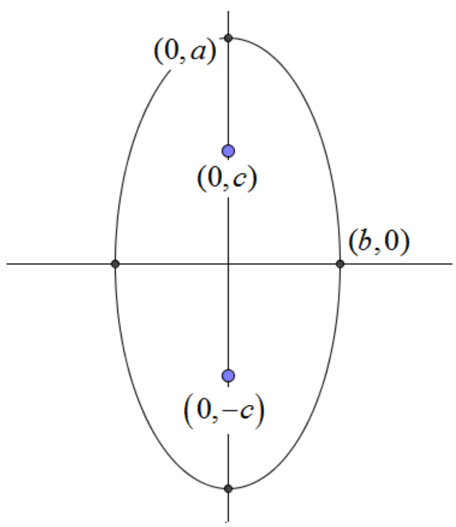
Horizontal Ellipse: $a \geq b$



$$\text{Formula: } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$c^2 = a^2 - b^2$$

Vertical Ellipse: $a \geq b$



$$\text{Formula: } \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$c^2 = a^2 - b^2$$

Example 3

Sketch $8x^2 + 36y^2 = 288$.

Need to divide by 288 on both sides to get it in the correct form to get

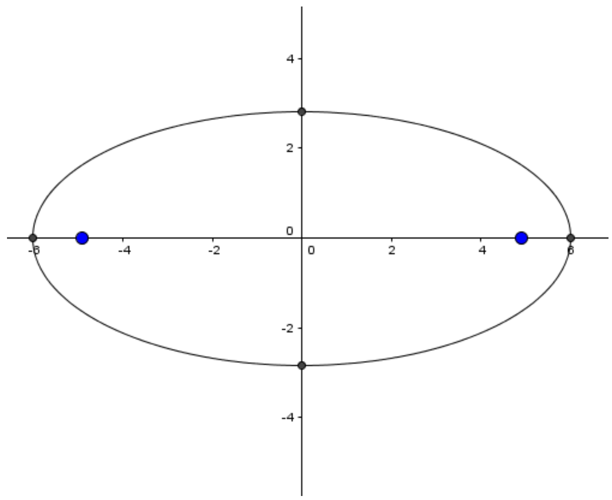
$$\frac{x^2}{36} + \frac{y^2}{8} = 1$$

From here we see that $a^2 = 36 \Rightarrow a = 6$ and $b^2 = 8 \Rightarrow b = \sqrt{8}$. To find c

$$c^2 = a^2 - b^2$$

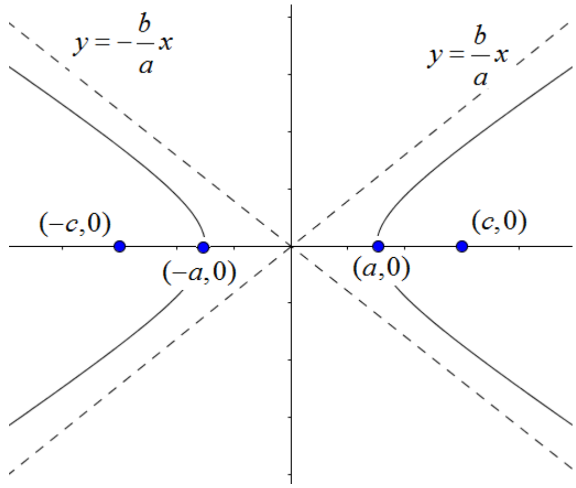
$$c^2 = 36 - 8 = 24$$

$$c = \sqrt{24}$$



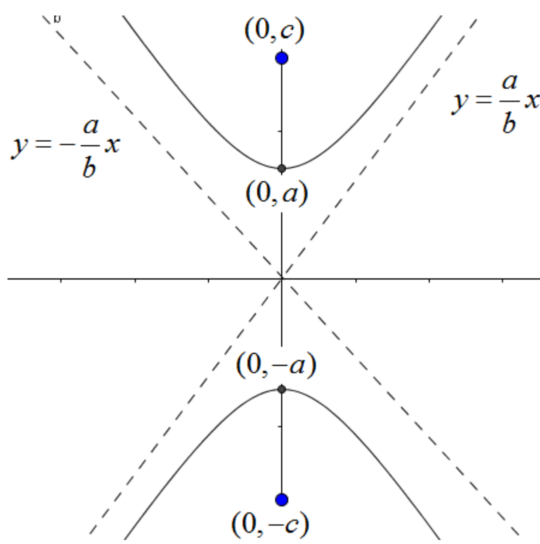
Definition 3: Hyperbola

A set of points in a plane such that the difference of whose distances from two fixed points is a constant.



$$\text{Formula: } \frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$$c^2=a^2+b^2$$



$$\text{Formula: } \frac{y^2}{a^2}-\frac{x^2}{b^2}=1$$

$$c^2=a^2+b^2$$

Example 4

Sketch $\dfrac{y^2}{9} - \dfrac{x^2}{4} = 1$

From the equation we see $a^2 = 9 \Rightarrow a = 3$ and $b^2 = 4 \Rightarrow b = 2$. Also $c^2 = a^2 + b^2 = 3^2 + 4^2 = 25$, so $c = 5$. Using the correct form we have the following graph