

Partial solutions for 7 8 11 12 13 21 22

7. $\nabla f(x, y) = \frac{1}{y}\mathbf{i} - \frac{x}{y^2}\mathbf{j}$, $\nabla f(2, 1) = \mathbf{i} - 2\mathbf{j}$, $D_{\mathbf{u}}f(2, 1) = -1$

8. $\nabla f(x, y) = 2x \ln y \mathbf{i} + (x^2/y)\mathbf{j}$, $\nabla f(3, 1) = 9\mathbf{j}$, $D_{\mathbf{u}}f(3, 1) = 108/3$

11. $\nabla f(x, y) = \langle e^x \sin(y), e^x \cos(y) \rangle$, $\nabla f(0, \pi/3) = \langle \frac{3}{2}, \frac{1}{2} \rangle$. Unit vector in direction of \mathbf{v} is $\frac{4 - 3\sqrt{3}}{10}$

12. $\langle \frac{y^2 - x^2}{(x^2 + y^2)^2}, \frac{-2xy}{(x^2 + y^2)^2} \rangle$, $\nabla f(1, 2) = \langle 3/25, -4/25 \rangle$, $D_{\mathbf{u}}f(1, 2) = -\frac{11}{25\sqrt{34}}$

13. $\nabla g(2, 4) = 2\mathbf{i} + \frac{1}{2}\mathbf{j}$, $D_{\mathbf{u}}g(2, 4) = \frac{7}{2\sqrt{5}}$

21. $\nabla f(x, y) = \langle \frac{2y}{\sqrt{x}}, 4\sqrt{x} \rangle$, $\nabla f(4, 1) = \langle 1, 8 \rangle$. Max rate of change $|\nabla f(4, 1)| = \sqrt{65}$

22. $\nabla = \langle t^2 e^{st}, (st + 1)e^{st} \rangle$. $\nabla f(0, 2) = \langle 4, 1 \rangle$. Max change is $|\nabla f(0, 2)| = \sqrt{17}$.