

Show all work to receive full credit.

1. (16 points) The acceleration of a football after being thrown due to air resistance and gravity is given by $a(t) = 1i - 2j - e^t k = \langle 1, -2, -e^t \rangle$. Suppose the initial velocity is $\vec{v}(0) = 0i + 50j + 25k$. Find the velocity function $v(\vec{t})$.

2. (8 points) Consider the function defined by $\vec{r}(t) = \langle 2 \sin(t), -3t, -2 \cos(t) \rangle$. Find the arc length of the curve over the interval $0 \leq t \leq 2$.

3. (8 points) Evaluate the limit

$$\lim_{(x,y) \rightarrow (0,0)} \frac{5x^2y}{3x^2 + 3y^2}$$

by first converting to polar coordinates.

4. (20 points) Consider the function $f(x, y) = e^{2xy} + y \cos(x)$.

(a) Find the linearization of $f(x, y)$ at the point $(0, 1, 1)$.

(b) Use (a) to approximate $f(0.1, 0.9)$.

5. (16 points) If $z = e^y \sin(x)$, where $x = t \ln s$ and $y = s^2 e^{t-1}$. Using the chain rule, find $\frac{\partial z}{\partial s}$ when $s = 1$ and $t = 1$.

6. (12 points) Let $f(x, y) = x^2 y^2 + xy - 1$.

(a) Find the gradient of f .

(b) Find the rate of change of f in the direction of $\vec{v} = \langle 3, 4 \rangle$ at the point $P(1, 0, -1)$.

7. (20 points) Find the local maximum, local minimum, and saddle points of the function
- $$f(x, y) = x^3 - y^2 - xy.$$