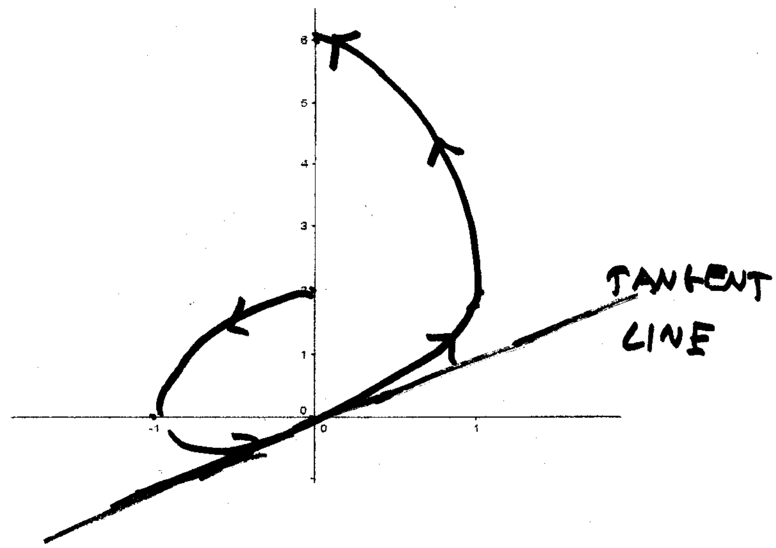
Exam 1-A Math 232, Calculus III, Fall 2016 Name: SOLUTION

Show all work for full credit.

1. Let $x = \sin(\pi t)$ and $y = 4t^2 + 2t$, $-1 \le t \le 1$

   (a) (10 points) Sketch the graph of the parametric equations on the interval $[-1, 1]$. Start with $t = -1$ and increase by $1/2$.

t	x	y
-1	0	2
$-1/2$	-1	0
0	0	0
$1/2$	1	2
1	0	6



   (b) (10 points) Find the equation of the tangent line at $t = 0$. Draw the line in the graph above.

(1) POINT AT $t=0$ IS $(0,0)$

(2) SLOPE: $\frac{dy}{dx} = \frac{8t+2}{\pi\cos(\pi t)}$, $\left.\frac{dy}{dx}\right|_{t=0} = \frac{8(0)+2}{\pi\cos 0} = \frac{2}{\pi}$

(3) LINE: $y - 0 = \frac{2}{\pi}(x-0)$

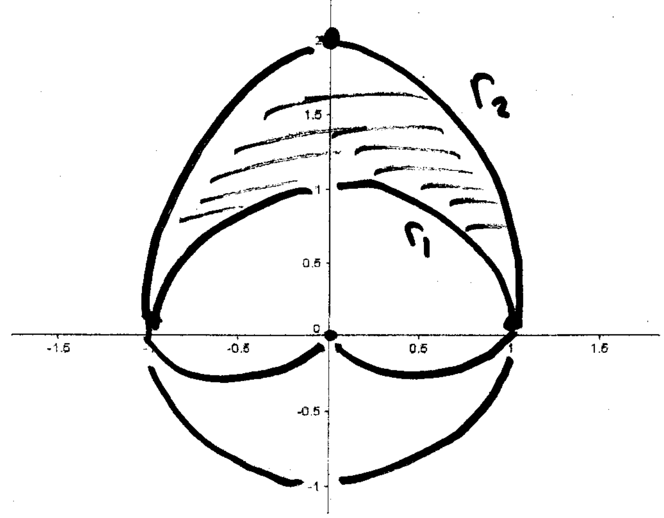
$$\boxed{y = \frac{2}{\pi}x}$$

   (c) (5 points) Set up the integral for the arc length.

$$\int_{-1}^{1} \sqrt{(8t+2)^2 + (\pi\cos\pi t)^2}\, dt$$

2. Let $r_1(\theta) = 1$ and $r_2(\theta) = 1 + \sin(\theta)$.

(a) (10 points) Sketch r_1 and r_2.



(b) (15 points) Evaluate the integral that represents the area inside $r_2 = 1 + \cancel{\cos(\theta)}$ $\sin(\theta)$ outside $r_1 = 1$ and

$$\int_0^{\pi} \frac{1}{2}(1+\sin\theta)^2 - \frac{1}{2}(1)^2 \, d\theta$$

$$= \frac{1}{2}\int_0^{\pi} 1 + 2\sin\theta + \sin^2\theta - 1 \, d\theta$$

$$= \frac{1}{2}\int_0^{\pi} 2\sin\theta + \frac{1}{2}(1 - \cos 2\theta)\, d\theta$$

$$= \int_0^{\pi} \sin\theta + \frac{1}{4} - \frac{1}{4}\cos 2\theta \, d\theta$$

$$= -\cos\theta + \frac{1}{4}\theta - \frac{1}{8}\sin 2\theta \Big|_0^{\pi}$$

$$= \left[-\cos\pi + \frac{1}{4}\pi - \frac{1}{8}\sin 2\pi\right] - \left[-\cos 0 + \frac{1}{4}(0) - \frac{1}{8}\sin 0\right]$$

$$= \left[1 + \frac{\pi}{4} - 0\right] - \left[-1 + 0 - 0\right]$$

$$= 2 + \frac{\pi}{4}$$

3. Let Let $P(3,1,2)$, $Q(6,0,5)$, and $R(8,9,0)$ be three points.

(a) (6 points) Find the vectors $\vec{a} = \vec{PQ}$ and $\vec{b} = \vec{PR}$

$$\vec{a} = \langle 6-3, 0-1, 5-2 \rangle = \langle 3, -1, 3 \rangle$$

$$\vec{b} = \langle 8-3, 9-1, 0-2 \rangle = \langle 5, 8, -2 \rangle$$

(b) (10 points) Find the vector equation, parametric equations, and symmetric equations of the line L through P and Q.

DIRECTION VECTOR FOR L IS $\vec{a} = \vec{PQ} = \langle 3, -1, 3 \rangle$

$r_0 = \langle 3, 1, 2 \rangle$

VECTOR: $r(t) = \langle 3, -1, 3 \rangle t + \langle 3, 1, 2 \rangle$

PARAMETRIC: $x = 3t + 3, \quad y = -1t + 1, \quad z = 3t + 2$

SYMMETRIC: $\dfrac{x-3}{3} = \dfrac{y-1}{-1} = \dfrac{z-2}{3}$

(c) (6 points) At what point does the line L intersect the yz-plane?

- MEANS $x = 0$: SOLVING $0 = 3t + 3$ GIVES $t = -1$
- PLUG $t = -1$ INTO $y = -t + 1$ AND $z = 3t + 2$

$y = 2 \qquad z = -1$

POINT $(0, 2, -1)$

(d) (6 points) Find $\vec{a} \times \vec{b}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 3 & -1 & 3 \\ 5 & 8 & -2 \end{vmatrix} = i \begin{vmatrix} -1 & 3 \\ 8 & -2 \end{vmatrix} - j \begin{vmatrix} 3 & 3 \\ 5 & -2 \end{vmatrix} + k \begin{vmatrix} 3 & -1 \\ 5 & 8 \end{vmatrix}$$

$$= i(2-24) - j(-6-15) + k(24 - -5)$$

$$= -22i + 21j + 29k$$

(e) (6 points) Are vectors $\vec{a}$ and $\vec{b}$ orthogonal, parallel, or neither? Explain.

NO $\rightarrow \perp$: $\langle 3,-1,3\rangle \cdot \langle 5,8,-2\rangle = 15-8-6 = 15-14=1$

NO $\rightarrow$ // SINCE $\vec{a}\times\vec{b} \neq 0$

ANSWER IS NEITHER

(f) (5 points) Find the area of the triangle formed by the points P, Q, and R.

$$\text{AREA} = \frac{1}{2}\left|\vec{PQ}\times\vec{PR}\right|$$
$$= \frac{1}{2}\left|\langle -22, 21, 29\rangle\right|$$
$$= \frac{1}{2}\sqrt{(-22)^2+(21)^2+(29)^2} = \frac{1}{2}\sqrt{1766}$$

(g) (6 points) Find the equation of the plane through the points $P(3, [illegible], 2)$, $Q(6, [illegible], 5)$, and $R(8, 9, 0)$

(1) NORMAL VECTOR TO PLANE IS $\vec{PQ}\times\vec{PR} = \langle -22, 21, 29\rangle$

(2) POINT $(3,0,2)$

(3) EQUATION $-22(x-3) + 21(y-[illegible]) + 29(z-2)$

(h) (5 points) Find the angle between the vectors $\vec{a}$ and $\vec{b}$.

$$\vec{a}\cdot\vec{b} = |a|\,|b|\cos\theta$$
$$1 = \sqrt{19}\,\sqrt{93}\,\cos\theta$$
$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{1767}}\right) \approx 89^\circ$$