QUIZ 9 MATH 230, CALCULUS II, SPRING 2017 NAME: SOLUTION

Show all work to receive full credit.

1. Given $\sum_{n=1}^{\infty} a_n = \frac{2}{3} + \frac{2}{7} + \frac{2}{11} + \frac{2}{15} + \frac{2}{19} + \frac{2}{23} + \ldots$.

(a) Find a_n.

$$a_n = \frac{2}{4n-1}$$

(b) Does a_n converge? If so, to what?

YES. $\lim_{n\to\infty} \frac{2}{4n-1} = 0$

(c) Find S_3

$$S_3 = \frac{2}{3} + \frac{2}{7} + \frac{2}{11} = \frac{262}{231} \approx 1.134$$

(d) Use the limit comparison test to determine whether the series converges. Explain your reasoning.

COMPARE TO $\sum \underbrace{\frac{2}{4n}}_{}^{b_n}$ OR $\frac{1}{2}\sum \frac{1}{n}$ WHICH DIVERGES BY P-TEST $(p=1)$

LCT: $\lim_{n\to\infty} \dfrac{\frac{2}{4n-1}}{\frac{2}{4n}} = \lim_{n\to\infty} \dfrac{8n}{8n-2} = 1$

$\sum_{n=1}^{\infty} \frac{2}{4n-1}$ DIVERGES BY LCT BECAUSE $\lim_{n\to\infty} \frac{a_n}{b_n} = 1$

AND $\sum \frac{2}{4n}$ DIVERGES BY P-TEST $(p=1)$

2. Determine if $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$ converges using

(a) Integral Test

$$\int_1^{\infty} \frac{e^{1/x}}{x^2}\,dx \quad \text{LET } u=\frac{1}{x},\ du=\frac{-1}{x^2}\,dx \Rightarrow \int -e^u\,du = -e^u$$

$$= -e^{1/x}\Big|_1^{\infty} = -e^0 + e^1 = e-1 \quad (\text{CONVERGENT})$$

$\sum \frac{e^{1/n}}{n^2}$ CONVERGES BY INTEGRAL TEST BECAUSE $\int_1^{\infty} \frac{e^{1/x}}{x^2}\,dx$ CONVERGES

(b) Direct Comparison Test

$\sum \frac{e^{1/n}}{n^2} \leq \sum \frac{e}{n^2}$ AND $\sum \frac{e}{n^2}$ CONVERGES BY p-TEST $p=2>1$

$\sum \frac{e^{1/n}}{n^2}$ CONVERGES BY DCT BECAUSE $\frac{e^{1/n}}{n^2} \leq \frac{e}{n^2}$ AND $\sum \frac{e}{n^2}$ CONV. BY p-TEST $(p=2>1)$