

Show all work to receive full credit. Form a group of FIVE students.

1. Evaluate  $\int_2^3 \frac{3x - 5x^2}{(3x-1)(x-1)^2} dx$ . Round answer to 5 decimals.

$$\text{DECOMP: } \frac{3x - 5x^2}{(3x-1)(x-1)^2} = \frac{A}{3x-1} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$\Rightarrow 3x - 5x^2 = A(x-1)^2 + B(3x-1)(x-1) + C(3x-1)$$

SHORTCUT:

$$x=1: \quad -2 = C(2) \rightarrow C = -1$$

$$x=1/3: \quad 4/9 = A(4/9) \rightarrow A = 1$$

$$\Rightarrow 3x - 5x^2 = Ax^2 - 2Ax + A + 3Bx^2 - 4Bx + B + 3Cx - C$$

$$3x - 5x^2 = (A+3B)x^2 + (-2A-4B+3C)x + (A+B-C)$$

$$\text{SO } \begin{aligned} A+3B &= -5 \\ -2A-4B+3C &= 3 \\ A+B-C &= 0 \end{aligned}$$

USE ANY OF THESE TO FIND B

$$A+B-C=0$$

$$1+B-(-1)=0$$

$$2+B=0$$

$$\boxed{B = -2}$$

$$\text{SETUP: } \int_2^3 \frac{1}{3x-1} + \frac{-2}{x-1} + \frac{-1}{(x-1)^2} dx$$

$$= \frac{1}{3} \ln|3x-1| - 2 \ln|x-1| + \frac{1}{x-1} \Big|_2^3$$

$$= \left( \frac{1}{3} \ln 8 - 2 \ln 2 + \frac{1}{2} \right) - \left( \frac{1}{3} \ln 5 - 2 \ln 1 + \frac{1}{1} \right)$$

$$= -1.72963$$

2. Approximate the integral from (1) using the following methods with  $n = 4$ . Identify  $\Delta x$ ,  $x_0, x_1, x_2, x_3, x_4$ , and write out the formula used. Round final answers to 5 decimals.

(a) Prep Area for  $\Delta x, x_0, x_1, x_2, x_3, x_4$  and  $f(x_0), f(x_1), \dots, f(x_4)$ .

$$\Delta x = \frac{3-2}{4} = \frac{1}{4}, \quad x_0 = 2, \quad x_1 = 2.25, \quad x_2 = 2.5, \quad x_3 = 2.75, \quad x_4 = 3$$

$$f(x_0) = -2.8$$

$$f(x_1) = -2.06609$$

$$f(x_2) = -1.62393$$

$$f(x_3) = -1.33146$$

$$f(x_4) = -1.125$$

(b) Lefthand Method

$$\begin{aligned} L_4 &= \Delta x [f(x_0) + f(x_1) + f(x_2) + f(x_3)] \\ &= \frac{1}{4} [-2.8 - 2.06609 - 1.62393 - 1.33146] \\ &= -1.95537 \end{aligned}$$

(c) Righthand Method

$$\begin{aligned} R_4 &= \Delta x [f(x_1) + f(x_2) + f(x_3) + f(x_4)] \\ &= \frac{1}{4} [-2.06609 - 1.62393 - 1.33146 - 1.125] \\ &= -1.53662 \end{aligned}$$

(d) Trapezoid Method

$$\begin{aligned}T_4 &= \frac{\Delta x}{2} [f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)] \\&= \frac{1/4}{2} [-2.8 + 2(-2.06609) + 2(-1.62393) + 2(-1.33146) - 1.125] \\&= -1.74600\end{aligned}$$

(e) Simpson's Method

$$\begin{aligned}S_4 &= \frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)] \\&= \frac{1/4}{3} [-2.8 + 4(-2.06609) + 2(-1.62393) + 4(-1.33146) - 1.125] \\&= -1.73026\end{aligned}$$