

Show all work to receive full credit.

1. Evaluate  $\int \frac{4}{x^2 \sqrt{4-x^2}} dx$

LET  $x = 2 \sin \theta$ ,  $dx = 2 \cos \theta d\theta$

$$\Rightarrow \int \frac{4}{(2 \sin \theta)^2 \sqrt{4 - (2 \sin \theta)^2}} \cdot 2 \cos \theta d\theta$$

$$= \int \frac{8 \cos \theta d\theta}{4 \sin^2 \theta \sqrt{4 - 4 \sin^2 \theta}}$$

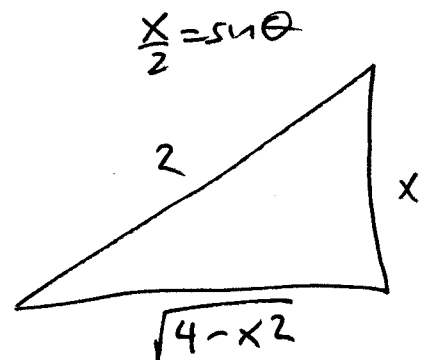
$$= \int \frac{8 \cos \theta d\theta}{4 \sin^2 \theta \sqrt{4 \cos^2 \theta}}$$

$$= \int \frac{8 \cos \theta d\theta}{8 \sin^2 \theta \cos \theta}$$

$$= \int \frac{d\theta}{\sin^2 \theta}$$

$$= \int \csc^2 \theta d\theta$$

$$= -\cot \theta + C$$



$$= -\cot \theta + C$$

$$= -\frac{\sqrt{4-x^2}}{x} + C$$

2. Evaluate  $\int \frac{3x - 5x^2}{(3x-1)(x-1)^2} dx$ .

$$(1) \quad \frac{3x - 5x^2}{(3x-1)(x-1)^2} = \frac{A}{3x-1} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$(2) \quad \begin{aligned} 3x - 5x^2 &= A(x-1)^2 + B(3x-1)(x-1) + C(3x-1) \\ &= A(x^2 - 2x + 1) + B(3x^2 - 4x + 1) + C(3x - 1) \\ &= Ax^2 - 2Ax + A + 3Bx^2 - 4Bx + B + 3Cx - C \\ &= (A + 3B)x^2 + (-2A - 4B + 3C)x + (A + B - C) \end{aligned}$$

★ SHORTCUT:

$$x=1: \quad -2 = c(3-1) \rightarrow -2 = 2c \quad c = -1$$

$$x=1/3: \quad \frac{4}{9} = A(1/3-1)^2 \rightarrow \frac{4}{9} = \frac{4}{9}A \rightarrow A=1$$

$$3x - 5x^2 = (A+B)x^2 + (-2A - 4B + 3C)x + (A+B-C)$$

FIND  
B

$$A + 3B = -5 \rightarrow \text{IF } A=1, B=-2$$

$$-2A - 4B + 3C = 3$$

$$A + B - C = 0$$

$$\int \frac{1}{3x-1} + \frac{-2}{x-1} + \frac{-1}{(x-1)^2} dx$$

$$= \frac{1}{3} \ln|3x-1| - 2 \ln|x-1| + \frac{1}{x-1} + C$$