QUIZ 5 MATH 230, CALCULUS II, SPRING 2017 NAME: SOLUTION

1. Evaluate $\lim_{x\to 0} \frac{e^{-2x}-1}{\sin^{-1} x}$ $= \frac{e^0-1}{\sin^{-1}0} = \frac{0}{0}$ DO LH RULE

$$\overset{LH}{=} \lim_{x\to 0} \frac{-2e^{-2x}}{\frac{1}{\sqrt{1-x^2}}} = \lim_{x\to 0} -2e^{-2x}\sqrt{1-x^2}$$

↑ SIMPLIFY

$$= -2e^0\sqrt{1-0^2}$$

$$= -2$$

2. Evaluate $\lim_{x\to 0^+} (1+\sin(3x))^{1/x}$

(1) REWRITE AS $e^{\frac{1}{x}\ln(1+\sin 3x)}$ AND EVALUATE $\lim_{x\to 0^+} \frac{1}{x}\ln(1+\sin 3x)$

(2) $$\lim_{x\to 0^+} \frac{\ln(1+\sin 3x)}{x} \underset{0/0}{\overset{LH}{=}} \lim_{x\to 0^+} \frac{\frac{1}{1+\sin 3x}\cdot 3\cos 3x}{1}$$

$$= \frac{\frac{1}{1+\sin 0}\cdot 3\cos 0}{1}$$

$$= 3$$

(3) FINAL: $\lim_{x\to 0^+} (1+\sin 3x)^{1/x} = e^3$

3. Evaluate $\int_0^{\pi/4} x \sec^2 x \, dx$.

(1) LET $u = x$, $dv = \sec^2 x dx$

$du = dx$, $v = \tan x$

BY PARTS (2) $x \tan x - \int \tan x \, dx = x \tan x - \ln|\sec x| \Big|_0^{\pi/4}$

$$= \left[\frac{\pi}{4} \tan \frac{\pi}{4} - \ln\Big|\underbrace{\sec \frac{\pi}{4}}_{\sqrt{2}}\Big|\right] - \left[\underbrace{0 \tan 0}_{0} - \underbrace{\ln|\sec 0|}_{\ln 1 = 0}\right] = \frac{\pi}{4} - \ln\sqrt{2}$$

4. Evaluate $\int x^2 e^{3x} \, dx$

(1) LET $u = x^2$, $dv = e^{3x} dx$

$du = 2x$, $v = \frac{1}{3} e^{3x}$

BY PARTS (2) $\frac{1}{3} x^2 e^{3x} - \int \frac{2}{3} x e^{3x} dx$

AGAIN (3) $\frac{1}{3} x^2 e^{3x} - \left[\begin{array}{ll} u = \frac{2}{3} x & dv = e^{3x} \\ du = \frac{2}{3} dx & v = \frac{1}{3} e^{3x} \end{array}\right]$

$$= \frac{1}{3} x^2 e^{3x} - \left[\frac{2}{9} x e^{3x} - \int \frac{2}{9} e^{3x} dx\right]$$

$$= \frac{1}{3} x^2 e^{3x} - \frac{2}{9} x e^{3x} + \frac{2}{27} e^{3x} + C$$