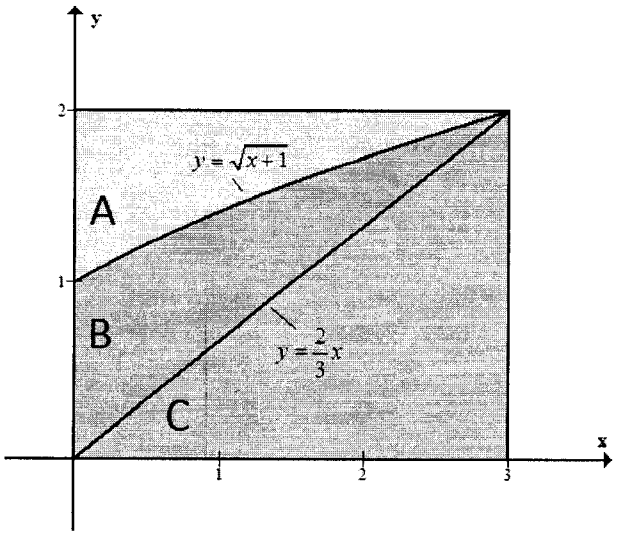
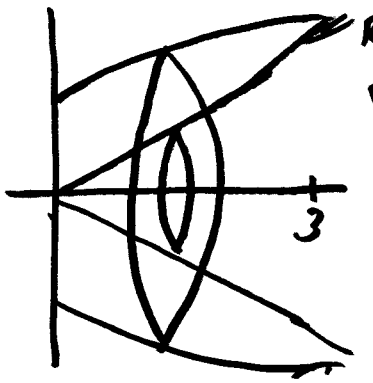
QUIZ 4 MATH 230, CALCULUS II, SPRING 2017 NAME: SOLUTION

1. Use the graph below to rotate the given region about the given axis of revolution. **Draw the picture and find the area.**

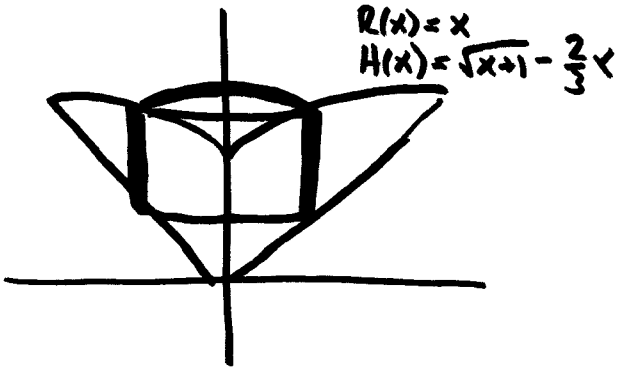


(a) Rotate region B about the x-axis. (Washer Method)



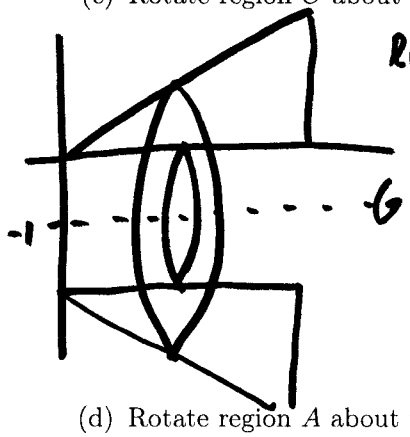
$$V=\int_0^3 \pi\left(\sqrt{x+1}\right)^2-\pi\left(\frac{2}{3}x\right)^2 dx$$
$$=\pi\int_0^3 x+1-\frac{4}{9}x^2$$
$$=\pi\left[\frac{1}{2}x^2+x-\frac{4}{27}x^3\Big|_0^3\right]$$
$$=\frac{7\pi}{2}$$

(b) Rotate region B about the y-axis (Using Shell Method).



$$V=\int_0^3 2\pi x\left(\sqrt{x+1}-\frac{2}{3}x\right)dx$$
$$=2\pi\int_0^3 x\sqrt{x+1}-\frac{2}{3}x^2\,dx$$
$$=2\pi\left[\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}-\frac{2}{9}x^3\Big|_0^3\right]$$
$$=2\pi\left[\left(\frac{22}{15}\right)-\left(\frac{-4}{15}\right)\right]$$
$$=\frac{52\pi}{15}$$

(c) Rotate region C about the line $y = -1$. (Washer Method)



$R(x) = \frac{2}{3}x+1$

$r(x) = 1$

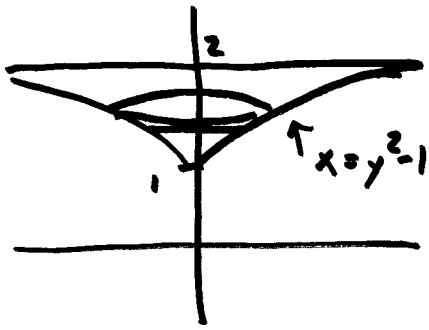
$V = \int_0^3 \pi\left(\frac{2}{3}x+1\right)^2 - \pi(1)^2\,dx$

$= \pi\int_0^3 \frac{4}{9}x^2 + \frac{4}{3}x\,dx$

$= \pi\left[\frac{4}{27}x^3 + \frac{4}{6}x^2\Big|_0^3\right]$

$= 10\pi$

(d) Rotate region A about the line y-axis (Disk Method).

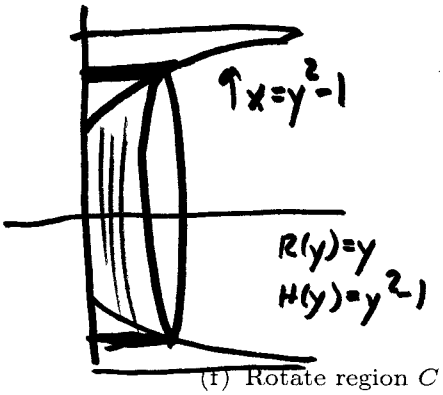


$V = \int_1^2 \pi(y^2-1)^2\,dy = \pi\int_1^2 y^4 - 2y^2 + 1\,dy$

$= \pi\left[\frac{1}{5}y^5 - \frac{2}{3}y^3 + y\Big|_1^2\right]$

$= \pi\left[\left(\frac{46}{15}\right) - \left(\frac{8}{15}\right)\right] = \frac{38\pi}{15}$

(e) Rotate region A around the line X-Axis (Shell).

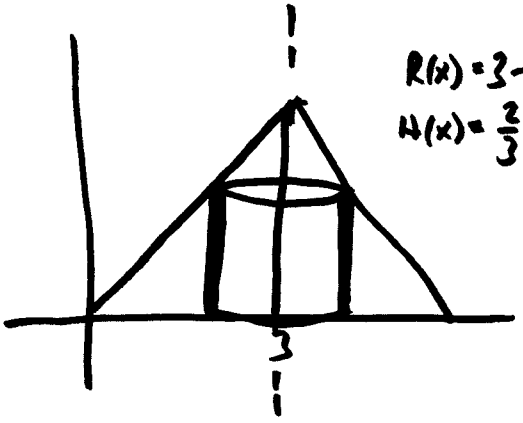


$V = \int_1^2 2\pi y(y^2-1)\,dy = 2\pi\int_1^2 y^3 - y\,dy$

$= 2\pi\left[\frac{1}{4}y^4 - \frac{1}{2}y^2\Big|_1^2\right]$

$= 2\pi\left[(2) - \left(-\frac{1}{4}\right)\right] = \frac{9\pi}{2}$

(f) Rotate region C around the line $x = 3$ (Shell).



$R(x) = 3-x$

$H(x) = \frac{2}{3}x$

$V = \int_0^3 2\pi(3-x)\left(\frac{2}{3}x\right)dx$

$= 2\pi\int_0^3 2x - \frac{2}{3}x^2\,dx$

$= 2\pi\left[x^2 - \frac{2}{9}x^3\Big|_0^3\right]$

$= 2\pi\left[(3) - (0)\right] = 6\pi$