

1. Differentiate the following functions (DO NOT SIMPLIFY FINAL ANSWER)

(a)  $f(x) = \ln(\tan x)$

$$f'(x) = \frac{1}{\tan x} \cdot \sec^2 x$$

(b)  $f(x) = \sin(e^{x^2})$

$$f'(x) = \cos(e^{x^2}) \cdot e^{x^2} \cdot 2x$$

(c)  $f(x) = \frac{x^4 \sqrt{3x-1}}{x-1}$  (Logarithmic Differentiation)

$$y = \frac{x^4 \sqrt{3x-1}}{x-1}$$

LOG BOTH SIDES  
 $\Rightarrow$

$$\ln y = \ln \left( \frac{x^4 \sqrt{3x-1}}{x-1} \right)$$

EXPAND  
 $\Rightarrow$

$$\begin{aligned} \ln y &= \ln(x^4) + \ln \sqrt{3x-1} - \ln(x-1) \\ \ln y &= 4 \ln x + \frac{1}{2} \ln(3x-1) - \ln(x-1) \end{aligned}$$

DIFFERENTIATE  
 BOTH SIDES  
 $\Rightarrow$

$$\frac{1}{y} \cdot y' = 4 \cdot \frac{1}{x} + \frac{1}{2} \cdot \frac{1}{3x-1} \cdot 3 - \frac{1}{x-1}$$

MULTIPLY BY  
 $y$

$$y' = y \left( \frac{4}{x} + \frac{3}{2(3x-1)} - \frac{1}{x-1} \right)$$

$$y' = \frac{x^4 \sqrt{3x-1}}{x-1} \left( \frac{4}{x} + \frac{3}{6x-2} - \frac{1}{x-1} \right)$$

(d)  $f(x) = e^{x^2 \cos(x)}$

$$f'(x) = e^{x^2 \cos x} \left( x^2(-\sin x) + 2x \cos x \right)$$

2. Evaluate the integral

(a)  $\int \frac{\tan(\ln x)}{x} dx$

(1) LET  $u = \ln x$

(2)  $du = \frac{1}{x} dx$

(3) SUBSTITUTE

$$\int \tan(u) du = \ln |\sec(u)| + C$$

$$= \ln |\sec(\ln x)| + C$$

(b)  $\int_0^1 e^x (1 - e^x)^2 dx$

(1) LET  $u = 1 - e^x$

(2)  $du = -e^x dx \rightarrow -du = e^x dx$

(3) CHANGE BOUNDS: IF  $x=1$ ,  $u = 1 - e^1 = 1 - e$   
IF  $x=0$ ,  $u = 1 - e^0 = 1 - 1 = 0$

(4) SUBSTITUTE

$$\int_0^{1-e} - (u)^2 du = -\frac{1}{3} u^3 \Big|_0^{1-e}$$

$$= -\frac{1}{3} (1-e)^3$$