QUIZ 10 MATH 230, CALCULUS II, SPRING 2017 NAME: SOLUTION

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1. Consider the convergent alternating series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n^2-1}$. Find the smallest value for n so that S_n has an error less than 0.03.

FIRST WAY: SOLVE $\frac{1}{2(n+1)^2-1} \leq .03$ FOR n

$$\frac{1}{2(n+1)^2-1} \leq .03$$

$$\frac{1}{.03} \leq 2(n+1)^2-1$$

$$34.33 \leq 2(n+1)^2$$

$$17.1667 \leq (n+1)^2$$

$$4.14 \leq n+1$$

$$3.14 \leq n$$

SO $\underline{n=4}$

SECOND WAY IS TRIAL AND ERROR:

$$b_{n+1} = \frac{1}{2(n+1)^2-1}$$

$n=1$ $b_2 = \frac{1}{2(2)^2-1} = .14$

$n=2$: $b_3 = \frac{1}{2(3)^2-1} = .059$

$n=3$: $b_4 = \frac{1}{2(4)^2-1} \approx .032$

$n=4$: $b_5 = \frac{1}{2(5)^2-1} \approx .02$ ✓

2. Determine which of the following series converge. If it converges, determine if it's conditional or absolute.

(a) $\sum_{n=1}^{\infty} \frac{3^n n^2}{(n+2)!}$ $\qquad a_n = \frac{3^n n^2}{(n+2)!} \qquad a_{n+1} = \frac{3^{n+1}(n+1)^2}{(n+3)!}$

ROOT: $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{3^{n+1}(n+1)^2}{(n+3)!} \cdot \frac{(n+2)!}{3^n n^2} \right| = \lim_{n\to\infty} \left| \frac{3^{n+1}}{3^n} \cdot \frac{(n+1)^2}{n^2} \cdot \frac{(n+2)!}{(n+3)!} \right|$

$= \lim_{n\to\infty} \left| 3 \cdot \frac{(n+1)^2}{n^2} \cdot \frac{1}{n+3} \right| = 3 \cdot 1 \cdot 0 = 0 < 1$

SO $\sum_{n=1}^{\infty} \frac{3^n n^2}{(n+2)!}$ CONVERGES ABSOLUTELY BY ROOT TEST SINCE $L = 0 < 1$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot n^2}{n^3 - 4}$

A.S.T LET $b_n = \frac{n^2}{n^3-4}$

(1) $b_n > 0$ ✓

(2) $b_n' = \frac{(n^3-4)\cdot 2n - n^2(3n^2)}{(n^3-4)^2}$

$= \frac{2n(-2n^3-4)}{(n^3-4)^2} < 0$

SO b_n IS DEC.

(3) $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{n^2}{n^3-4} = 0$ ✓

(1), (2), (3) SHOWS $\sum \frac{(-1)^n n^2}{n^3-4}$ CONV. BY A.S.T

CHECK FOR ABSOLUTE: $\sum \left| \frac{(-1)^n n^2}{(n^3-4)} \right| = \sum \frac{n^2}{n^3-4}$

$\frac{n^2}{n^3-4} \geq \frac{n^2}{n^3} = \frac{1}{n}$ AND $\sum \frac{1}{n}$ DIV. BY P-TEST $p=1$

SO $\sum \frac{n^2}{n^3-4}$ DIV. BY D.C.T

CONCLUSION: $\sum \frac{(-1)^n n^2}{n^3-4}$ CONVERGES CONDITIONALLY

3. $\sum_{n=1}^{\infty} \left(\frac{n+1}{n}\right)^n$. Explain why the Root Test cannot be used for this series.

$\lim_{n\to\infty} \sqrt[n]{\left(\frac{n+1}{n}\right)^n} = \lim_{n\to\infty} \frac{n+1}{n} = 1$. THE TEST IS INCONCLUSIVE