Exam 2A Solutions

Math 230, Fall 2015

Name:

Directions: Show ALL work for full credit. No calculators allowed.

1. True or False: The partial fraction decomposition for the following functions are as given below. If false, justify your reasoning.(12 pts)

a) $\dfrac{x(x^2+4)}{x^2-4} = \dfrac{A}{x+2} + \dfrac{B}{x-2}$ T F

The correct answer is false. This fraction is not proper, so we must long divide first.

b) $\dfrac{x^2+4}{x(x^2-4)} = \dfrac{A}{x} + \dfrac{B}{x+2} + \dfrac{C}{x-2}$ T F

The correct answer is true.

c) $\dfrac{x^2+4}{x^2(x-4)} = \dfrac{A}{x^2} + \dfrac{B}{x-4}$ T F

The correct answer is false. x^2 is a repeated linear factor. The correct partial fractions decomposition is $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-4}$

2. Set up integration by parts for the integral $\int z^3 e^z dz$, what you should choose for u and dv? Explain your answer. (10 pts)

For this integral, let $u = z^3$ and $dv = e^z dz$, with $du = 3z^2 dz$ and $v = e^z$. This is the best choice for u and dv since the power on the z variable will eventually decrease and eventually you will integrate one. I also excepted answers with "LIATE" as long as the words you wrote matched somewhat to mine above.

3. Calculate the limits. (21 pts)

a) $\lim_{x\to 0} \frac{\sin^{-1}(x)}{x}$

By plugging in 0, we have the form 0/0 and LH may be used.

$$\lim_{x\to 0} \frac{\sin^{-1}(x)}{x} = \lim_{x\to 0} \frac{1/\sqrt{1-x^2}}{1} = \lim_{x\to 0} \frac{1}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-0^2}} = 1$$

b) $\lim_{x\to\infty} \frac{\ln(2+e^x)}{3x}$

By plugging in ∞ we have the form ∞/∞ and LH may be used.

$$\lim_{x\to\infty} \frac{\ln(2+e^x)}{3x} = \lim_{x\to\infty} \frac{\frac{e^x}{2+e^x}}{3} = \lim_{x\to\infty} \frac{e^x}{6+3e^x}$$

Noticing that this last limit has form ∞/∞, LH may be used one more time to get

$$\lim_{x\to\infty} \frac{e^x}{3e^x} = \lim_{x\to\infty} \frac{1}{3} = \frac{1}{3}$$

c) $\lim_{x\to\infty} x^{1/x}$

Re-writing the function as $e^{\ln(x^{1/x})} = e^{1/x\ln(x)}$, we have

$$\lim_{x\to\infty} x^{1/x} = \lim_{x\to\infty} e^{1/x\ln(x)} = e^{\lim_{x\to\infty} 1/x\ln(x)}$$

Evaluating the limit in the exponent of e and using LH on that, we get

$$\lim_{x\to\infty} 1/x\ln(x) = \lim_{x\to\infty} \frac{\ln(x)}{x} = \lim_{x\to\infty} \frac{1}{x} = 0$$

But remember, our limit was

$$e^{\lim_{x\to\infty} 1/x\ln(x)} = e^0 = 1$$

4. Calculate the derivative of $f(x) = 3^{x^2}\tan^{-1}(x^2)$. (7 pts)

$$f'(x) = 3^{x^2}\frac{2x}{1+x^4} + \tan^{-1}(x^2)3^{x^2}\ln(3)(2x)$$

5. Evaluate the integral (the best 5 will be graded). (50 pts)

a) $\displaystyle\int \frac{dx}{1+9x^2}$

Using u sub, we get $u = 3x$, $du = 3dx$ and

$$\int \frac{dx}{1+9x^2} = \frac{1}{3}\int \frac{1}{1+u^2}du = \frac{1}{3}\tan^{-1}(u) + C = \frac{1}{3}\tan^{-1}(3x) + C$$

b) $\displaystyle\int \frac{x+2}{x^2+3x-4}dx$

Using partial fractions, we get

$$\int \frac{x+2}{x^2+3x-4}dx = \int \frac{2/5}{x+4} + \frac{3/5}{x-1}dx = 2/5\ln|x+4| + 3/5\ln|x-1| + C$$

c) $\displaystyle\int \tan^2(\theta)\sec^4(\theta)d\theta$

Re-writing this integral as

$$\int \tan^2(\theta)\sec^4(\theta)d\theta = \int \tan^2(\theta)\sec^2(\theta)\sec^2(\theta)d\theta = \int \tan^2(\theta)(1+\tan^2(\theta))\sec^2(\theta)d\theta$$

we see that by letting $u = \tan(\theta)$ and $du = \sec^2(\theta)d\theta$, we have

$$\int \tan^2(\theta)(1+\tan^2(\theta))\sec^2(\theta)d\theta = \int u^2(1+u^2)du = \int u^2+u^4du = 1/3u^3+1/5u^5+C = 1/3\tan^3(\theta)+1/5\tan$$

d) $\displaystyle\int 3^{\sin(\theta)}\cos(\theta)d\theta$

Letting $u = \sin(\theta)$, we have $du = \cos(\theta)d\theta$ which makes our integral

$$\int 3^{\sin(\theta)}\cos(\theta)d\theta = \int 3^u du = \frac{3^u}{\ln(3)} + C = \frac{3^{\sin(\theta)}}{\ln(3)} + C$$

e) $\int \frac{1}{x^2\sqrt{9-x^2}}dx$

This is a trig sub integral with $x = 3\sin(\theta)$ and $dx = 3\cos(\theta)d\theta$. Then our integral becomes

$$\int \frac{1}{x^2\sqrt{9-x^2}}dx = \int \frac{1}{(3\sin(\theta))^2\sqrt{9-(3\sin(\theta))^2}}3\cos(\theta)d\theta = \int \frac{1}{9\sin^2(\theta)\sqrt{9-9\sin^2(\theta)}}3\cos(\theta)d\theta$$

$$= \int \frac{1}{9\sin^2(\theta)\sqrt{9(1-\sin^2(\theta))}}3\cos(\theta)d\theta = \int \frac{1}{9\sin^2(\theta)\sqrt{9\cos^2(\theta)}}3\cos(\theta)d\theta$$

$$= \int \frac{1}{9\sin^2(\theta)3\cos(\theta)}3\cos(\theta)d\theta = \int \frac{1}{9\sin^2(\theta)}d\theta = \frac{1}{9}\int \csc^2(\theta)d\theta = -\frac{1}{9}\cot(\theta) + C$$

Now using a reference triangle, we see that $\cot(\theta) = \frac{\sqrt{9-x^2}}{x}$, and we have our answer of $-\frac{\sqrt{9-x^2}}{9x} + C$.

f) $\int \ln(x+3)dx$

There are two ways to evaluate this integral. Here is the easiest. Let $t = x+3$ and $dt = dx$. Then

$$\int \ln(x+3)dx = \int \ln(t)dt$$

This integral may be solved using integration by parts. Let $u = \ln(t)$ and $dv = dt$. Then $du = \frac{1}{t}dt$ and $v = t$.

$$\int \ln(t)dt = t\ln(t) - \int \frac{1}{t}tdt = t\ln(t) - \int 1dt = t\ln(t) - t + C = (x+3)\ln|x+3| - (x+3) + C$$

Extra Credit Prove that $e = \lim_{n\to\infty}(1+\frac{1}{n})^n$.

This was done in class. Re-write the limit as

$$\lim_{n\to\infty}(1+\frac{1}{n})^n = \lim_{n\to\infty} e^{\ln((1+\frac{1}{n})^n)} = \lim_{n\to\infty} e^{n\ln(1+\frac{1}{n})} = e^{\lim_{n\to\infty} n\ln(1+\frac{1}{n})}$$

Now just consider evaluating the limit in the exponent.

$$\lim_{n\to\infty} n\ln(1+\frac{1}{n}) = \lim_{n\to\infty}\frac{\ln(1+\frac{1}{n})}{1/n}$$

By plugging in ∞, we get $0/0$ and we can use LH.

$$\lim_{n\to\infty}\frac{\ln(1+\frac{1}{n})}{1/n} = \lim_{n\to\infty}\frac{(\frac{1}{1+1/n})(-1/n^2)}{-1/n^2} = \lim_{n\to\infty}\frac{1}{1+\frac{1}{n}} = 1$$

But remember we were evaluating

$$e^{\lim_{n\to\infty} n\ln(1+\frac{1}{n})} = e^1 = e$$